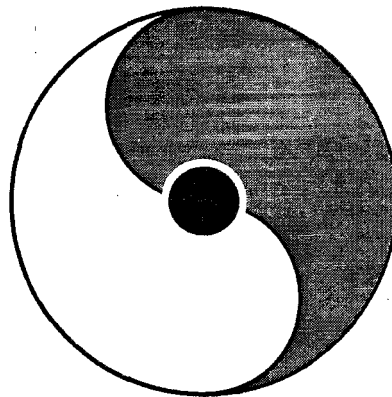


COLOR SUPERCONDUCTIVITY, INSTANTONS AND PARITY (NON?)-CONSERVATION AT HIGH BARYON DENSITY

November 11, 1997



Organizer

Miklos Gyulassy

RIKEN BNL Research Center

Building 510, Brookhaven National Laboratory, Upton, NY 11973, USA

Other RIKEN BNL Research Center Proceedings Volumes:

Volume 1 - Open Standards for Cascade Models for RHIC - BNL-64722

June 23-27, 1997 - Organizer - Miklos Gyulassy

Volume 2 - Perturbative QCD as a Probe of Hadron Structure - BNL-64723

July 14-25, 1997 - Organizers Robert Jaffe and George Sterman

Volume 3 - Hadron Spin-Flip at RHIC Energies - BNL-64724

July 21 - August 22, 1997 - Organizers T.L. Trueman and Elliot Leader

Volume 4 - Inauguration Ceremony, September 22 and

Non-Equilibrium Many Body Dynamics - BNL-64912

September 23-25, 1997, Organizer: M. Gyulassy

Preface to the Series

The RIKEN BNL Research Center was established this April at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including hard QCD/spin physics, lattice QCD and RHIC physics through nurturing of a new generation of young physicists.

For the first year, the Center will have only a Theory Group, with an Experimental Group to be structured later. The Theory Group will consist of about 12-15 Postdocs and Fellows, and plans to have an active Visiting Scientist program. A 0.6 teraflop parallel processor will be completed at the Center by the end of this year. In addition, the Center organizes workshops centered on specific problems in strong interactions.

Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form a proceedings, which can therefore be available within a short time.

T.D. Lee
July 4, 1997

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Workshop on Color Superconductivity, Instantons and
Parity (Non?)-Conservation at High Baryon Density

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Color Super-conductivity, Instantons, and Discrete Symmetry Breaking at High Baryon Density

This one day Riken BNL Research Center workshop was organized to follow-up on the rapidly developing theoretical work on color super-conductivity, instanton dynamics, and possible signatures of parity violation in strong interactions that was stimulated by the talk of Frank Wilczek during the Riken BNL September Symposium. The workshop was held on November 11, 1997 at the center with over 30 participants. The program consisted of four talks on theory in the morning followed by two talks in the afternoon by experimentalists and open discussion. Krishna Rajagopal (MIT) first reviewed the status of the chiral condensate calculations at high baryon density within the instanton model and the percolation transition at moderate densities restoring chiral symmetry. Mark Alford (Princeton) then discussed the nature of the novel color super-conducting diquark condensates. The main result was that the largest gap on the order of 100 MeV was found for the 0^+ condensate, with only a tiny gap \ll MeV for the other possible 1^+ . Thomas Schaefer (INT) gave a complete overview of the instanton effects on correlators and showed independent calculations in collaboration with Shuryak (SUNY) and Velkovsky (BNL) confirming the updated results of the Wilczek group (Princeton, MIT). Yang Pang (Columbia) addressed the general question of how breaking of discrete symmetries by any condensate with suitable quantum numbers could be searched for experimentally especially at the AGS through longitudinal Λ polarization measurements. Nicholas Samios (BNL) reviewed the history of measurements on Λ polarization and suggested specific kinematical variables for such analysis. Brian Cole (Columbia) showed recent E910 measurements of Λ production at the AGS in nuclear collisions and focused on the systematic biases that must be considered when looking for small symmetry breaking effects. Lively discussions led by Robert Jaffe (MIT) focused especially on speculations on the still unknown signatures of 0^+ color super-conductivity which of course would not be observable via discrete symmetry breaking.

Thanks to Brookhaven National Laboratory and to the U.S. Department of Energy for providing the facilities to hold this Workshop.

Miklos Gyulassy (Organizer)

QCD at Finite Baryon Density: Nucleon Droplets and Color Superconductivity

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We use a variational procedure to study finite density QCD in an approximation in which the interaction between quarks is modelled by that induced by instantons. We find that any uniform state of nonzero density with conventional chiral symmetry breaking has negative pressure with respect to empty space, and is therefore unstable. This is a precisely defined phenomenon which motivates the basic picture of hadrons assumed in the MIT bag model, with nucleons as droplets of chiral symmetry restored phase. At all densities high enough that the chirally symmetric phase fills space, we find that color symmetry is broken by the formation of a $\langle qq \rangle$ condensate of quark Cooper pairs. A plausible ordering scheme leads to a substantial gap in a Lorentz scalar channel involving quarks of two colors, and a much smaller gap in an axial vector channel involving quarks of the third color.

QCD AT FINITE

BARYON DENSITY:

NUCLEON DROPLETS

&

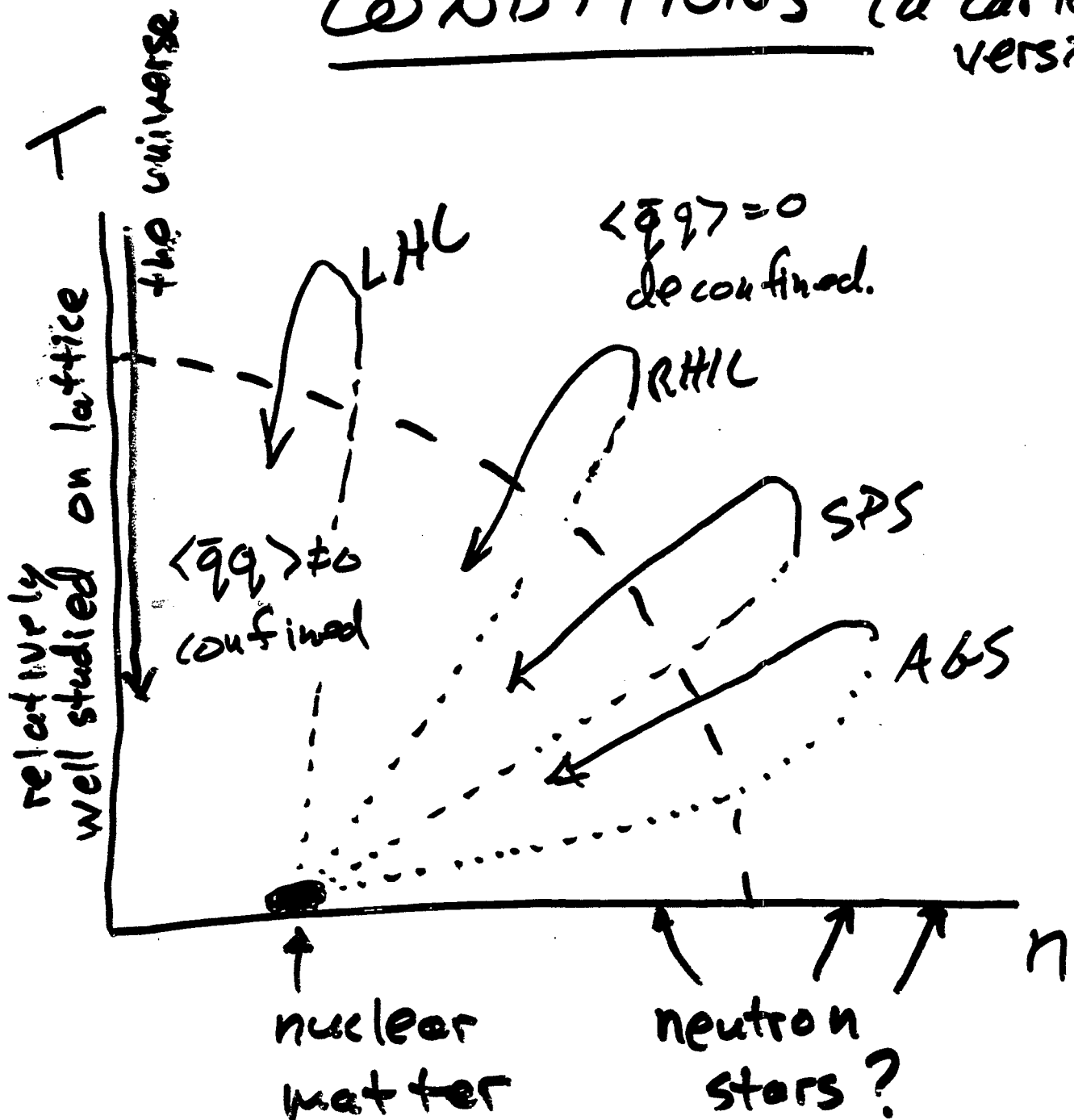
COLOR SUPERCONDUCTIVITY

Krishna Rajagopal

with Mark Alford,
Frank Wilczek.

Work in Progress

QCD IN EXTREME CONDITIONS (a cartoon version)



We'll redraw this cartoon
at the end of the talk.

How might we treat QCD at finite density?

6

x lattice

Monte Carlo requires positive measure

x SUSY

statistics matter

?? $N \gg 1$

- would have to take $N \rightarrow \infty$ with N odd.
- also, $N=3$ matters in the following.

perturbative

- valid at high enough density
- leads us to expect:
 - i) Chiral symmetry restored
 - ii) deconfinement
 - iii) color superconductivity.

variational

Not systematic, but can be applied at finite ($\neq 0$) density.

COLOR SUPERCONDUCTIVITY

AT $n \rightarrow \infty$

Bailin & Love

asymptotic freedom \rightarrow quarks
at Fermi surface (which is
large) are weakly interacting.

\rightarrow one gluon exchange

BCS: Any attractive interaction,
no matter how weak, \rightarrow

Cooper pairs, i.e. $\langle q, q \rangle$

One gluon exchange is attractive
in color 3 channel. $\Rightarrow \langle q^a q^b \epsilon_{abx} \rangle$
 $\neq 0$

Breaks $SU(3)_{\text{color}} \rightarrow SU(2)$
by Higgs mechanism.

$$(SU(3) \times U(1)_{EM} \rightarrow SU(2) \times U(1)_{\text{new}})$$

OUTLINE

The model

Chiral Symmetry Breaking
& restoration

Color Superconductivity

What was left out.

THE MODEL

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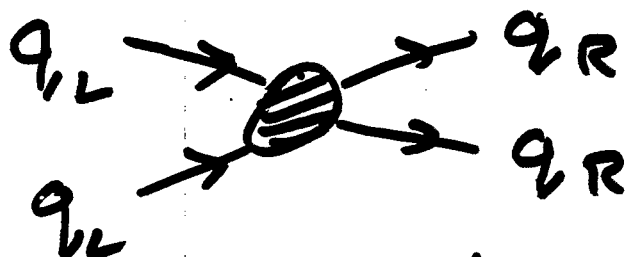
$H = H_{\text{kinetic}} \leftarrow \text{as for free quarks}$
 $+ H_I \leftarrow \text{4-fermion interaction.}$

History: BCS \rightarrow NJL
 $\quad \quad \quad \nwarrow \quad \nearrow$
 $\quad \quad \quad u_s$

What H_I ??

We work in 2-flavor QCD
with massless m_u, m_d .

Take H_I from instanton
interaction in QCD



Breaks all symmetries QCD
breaks. CNB including $(\bar{\psi}\psi)_A$

You should feel free to redo our
analyses adding other operators to H_I .

$$H_I = -K \int d^3x \bar{\Psi}_{R1\alpha} \Psi_{Lk}^{\gamma} \bar{\Psi}_{R2\beta} \Psi_{L\ell}^{\delta} \\ \times \xi^{k\ell} \times (3 \delta_{\gamma}^{\alpha} \delta_{\delta}^{\beta} - \delta_{\delta}^{\alpha} \delta_{\gamma}^{\beta})$$

$1, 2, k, \ell$: flavor; $\in \{1, 2\}$ + c.c.

$\alpha, \beta, \gamma, \delta$: color; $\in \{1, 2, 3\}$

Indices come from 't Hooft's analysis of QCD instanton.

Need to implement physics of asymptotic freedom:

In momentum space, multiply by a form factor

$$F(p) = \left(\frac{\Lambda^2}{\Lambda^2 + p^2} \right)^{\nu}$$

for each fermion.

3 parameters: $K \leftarrow$ will be fixed

$\Lambda \leftarrow$ not a cutoff.

$\nu \leftarrow$ we have used $\nu = 1/2, 1$.

We have explored $300 \text{ MeV} < \Lambda < 1 \text{ GeV}$

CHIRAL SYMMETRY

BREAKING ALONE (no S.C.)

Begin with ground state. (ie zero density.)

We choose a variational wave function:

$$|\Psi\rangle = \prod_{p,i,\alpha} \left[\cos \Theta^L(p) + \sin \Theta^L(p) a_{Li\alpha}^\dagger(p) b_{Ri\alpha}^\dagger(-p) \right] \\ \times \left[\cos \Theta^R(p) - \sin \Theta^R(p) a_{Ri\alpha}^\dagger b_{Li\alpha}^\dagger(-p) \right] |0\rangle$$

a^\dagger, b^\dagger : create particles & antiparticles

Θ : to be found by minimizing $\langle \Psi | H - \mu N | \Psi \rangle$

Pairing between quarks & antiquarks with same flavor & color, opposite helicity.

Varying $\langle \psi | H | \psi \rangle$ w.r.t. the θ 's

$$\rightarrow \tan(2\theta^L(p)) = \tan(2\theta^R(p)) = \frac{F^2(p) \Delta}{p}$$

where

$$\Delta \equiv 16 K \int_0^\infty \frac{p^2 dp}{2\pi^2} F^2(p) \sin \theta(p) \cos \theta(p)$$

Self consistency \Leftrightarrow

$$\Delta \cdot 1 = 8 K \int_0^\infty \frac{p^2 dp}{2\pi^2} \frac{F^4(p) \cdot \Delta}{\sqrt{F^4(p) \Delta^2 + p^2}}$$

GAP EQUATION

Solve; set $\Delta = 300 \text{ MeV}$. This fixes K as a function of the parameters Λ, ν .

NB: $\Delta = 0$ if K too small.

NONZERO DENSITY

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Trial wave function:

$$|\psi\rangle = \pi \left[\dots a_L^\dagger b_R^\dagger \right] \left[a_R^\dagger b_L^\dagger \right] |P_F\rangle$$

Where P_F has all particle states with $|p| < P_F$ filled. Antiparticle states of course empty.

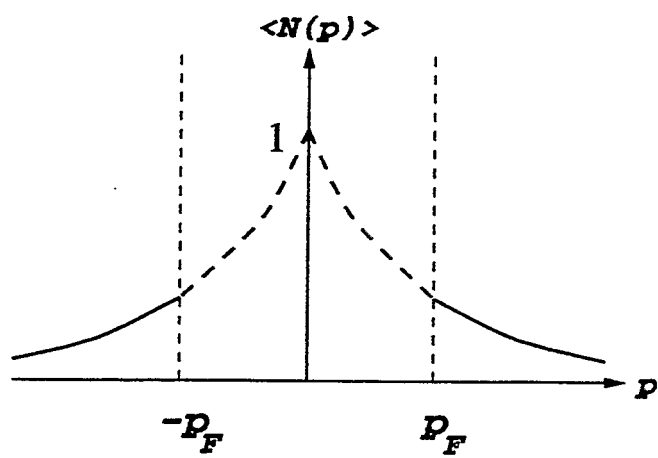
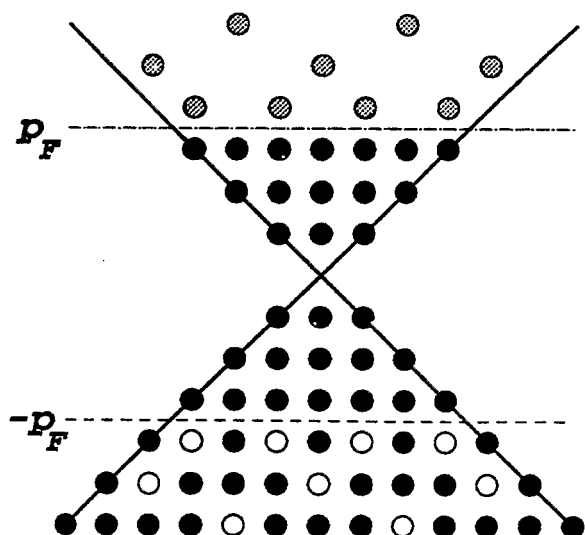
$$n = \frac{2}{\pi^2} P_F^3 \quad ; P_F \neq \mu$$

- Condensate does not affect n .
 $\rightarrow \theta(p) = 0$ for $|p| < P_F$.

Gap equation becomes:

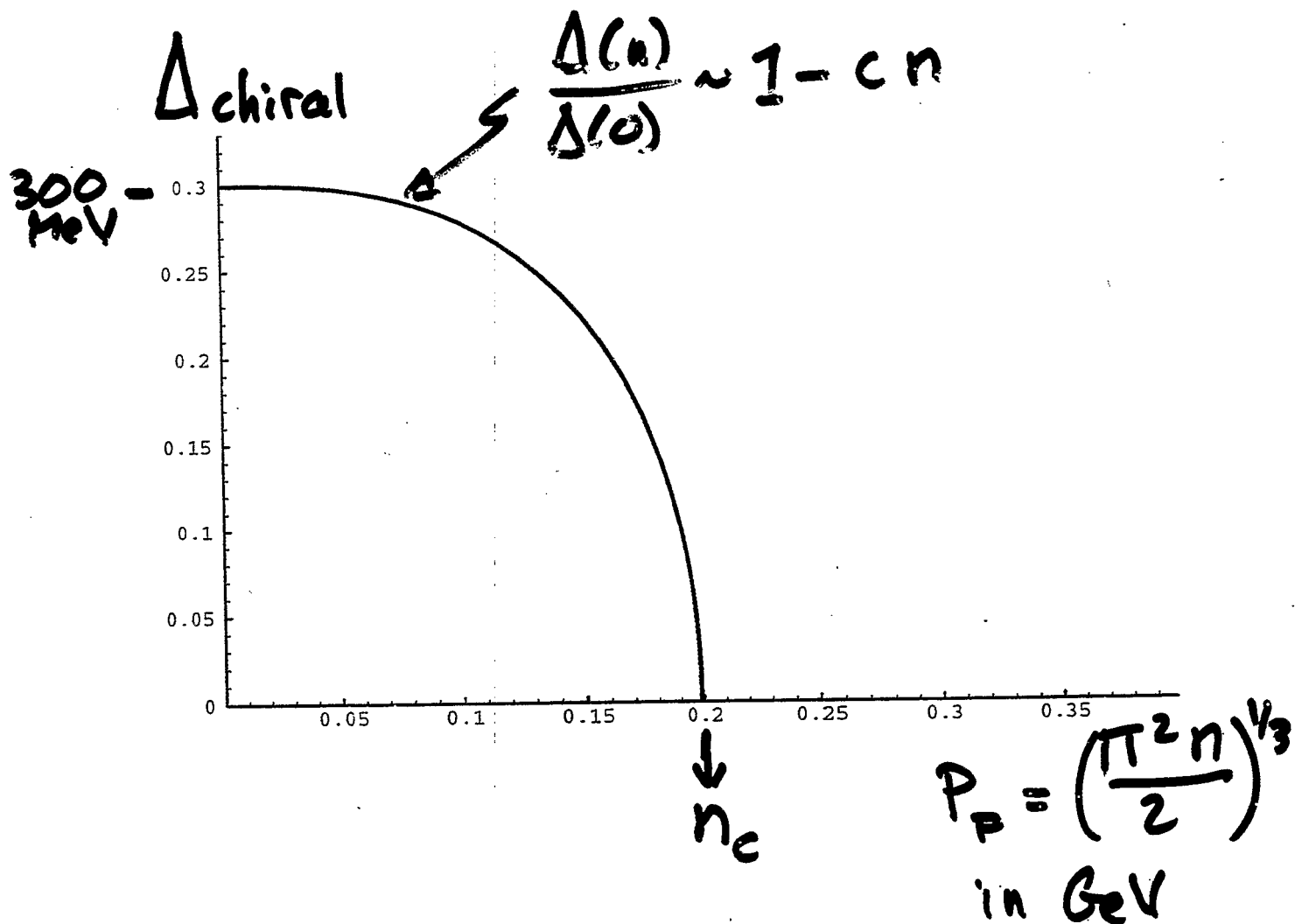
$$1 = 8K \int_{P_F}^{\infty} \frac{p^2 dp}{2\pi^2} \frac{F^4}{\sqrt{F^4 \Delta^2 + p^2}}$$

$\rightarrow \Delta$ decreases as n increases



CHIRAL GAP

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$$\Lambda = 700 \text{ MeV}$$

$$\nu = 1$$

With wave function in hand,
we can obtain:

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i) Energy density

$$\frac{\mathcal{E}}{24} = \frac{P_F^4}{16\pi^2} + \int_{P_F}^{\infty} \frac{p^2 dp}{2\pi^2} \frac{p}{2} \left(1 - \frac{p}{\sqrt{\Delta^2 F^4(p) + p^2}} \right) - \frac{\Delta^2}{32K}$$

ii) From \mathcal{E} & n we can
obtain

$$\mu \equiv \left. \frac{\partial \mathcal{E}}{\partial n} \right|_V$$

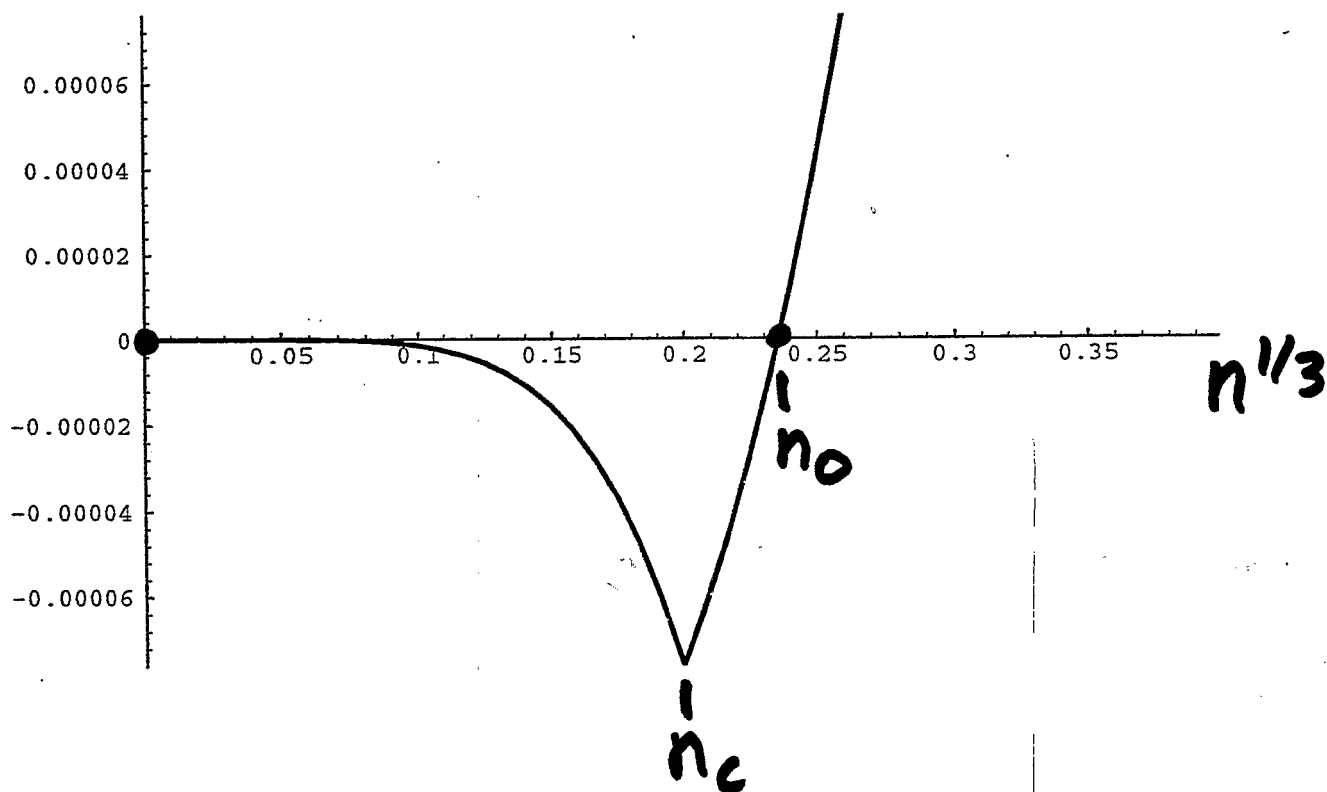
and find $\mu^2 = P_F^2 + \underbrace{\Delta^2 F^4(P_F)}_{(\text{gap})^2}$

NB μ decreases with n .

iii) Pressure = $n \frac{\partial \mathcal{E}}{\partial n} - \mathcal{E}$

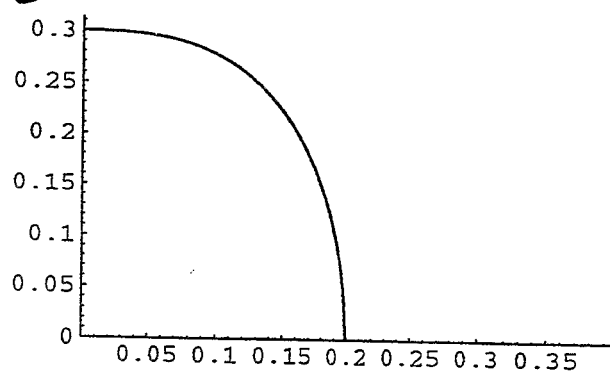
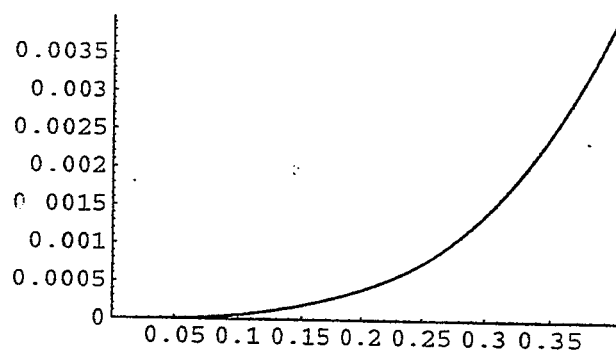
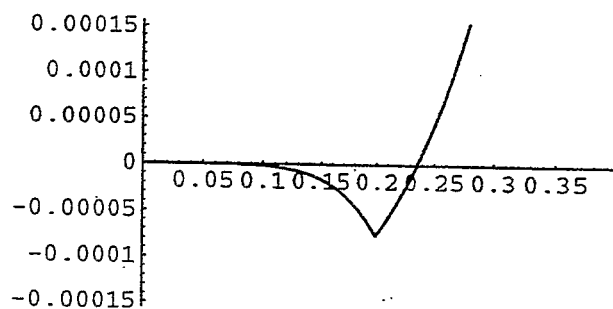
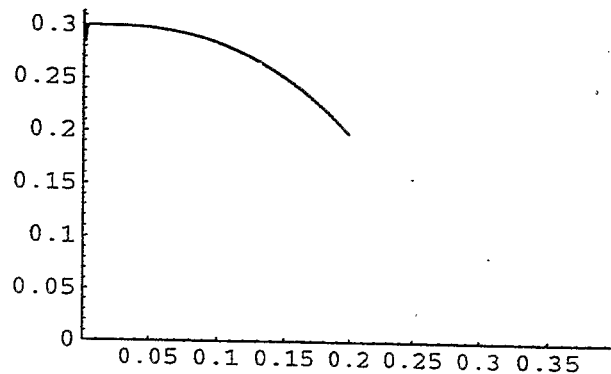


PRESSURE



$P < 0$ for all n with $\Delta \neq 0!$

Stable, zero pressure,
phases at $n = 0$
 $n = n_0 > n_c$

Δ  ϵ  P  μ 

IMPLICATIONS OF $P < 0$

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Uniform χ SB phase is
MECHANICALLY UNSTABLE at
 $n > 0$.

Not surprising at $n \rightarrow 0$.
Interesting thing: instability for
all n with $\Delta \neq 0$.

→ Break up into droplets of
 $n = n_0$ phase, with $\Delta = 0$
surrounded by vacuum,
with $n = 0$, $\Delta \neq 0$.

We cannot follow the breakup
into droplets, and stabilization,
with our present treatment.

However, presumably,

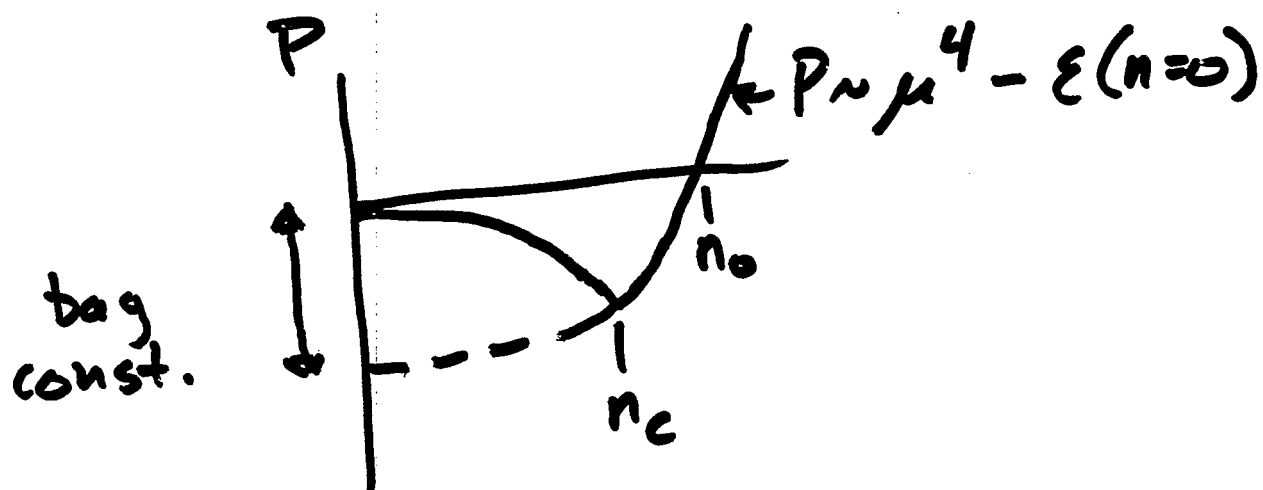
DROPLETS = NUCLEONS.

As in MIT bag model.

Get bag constant $\epsilon(n=0)$.



What is robust is the negative pressure. Our model is incomplete - no gluons, for example. Adding degrees of freedom probably required to get $\epsilon(n=0)$ & n_0 - related - to be larger.



PARAMETER DEPENDENCE

ν	Λ	$(\frac{\pi^2}{2} n_c)^{1/3}$	$(\frac{\pi^2}{2} n_0)^{1/3}$	$\xi(n=0)$
1	.3	.127	.170	$(.081)^4$
	.5	.168	.209	$(.100)^4$
	.7	.199	.235	$(.112)^4$
$1/2$.3	.161	.201	$(.096)^4$
	.5	.206	.241	$(.114)^4$
	.7	.239	.266	$(.126)^4$

(All in GeV)

n.b:

$$\frac{\pi^2}{2} n = p_F$$

Want to identify this with internal density in a nucleon. (~ 3 times nuclear matter density)
BUT That is $\sim .39$ in these units. \Rightarrow our n_0 too low

not crazy, but a little low.

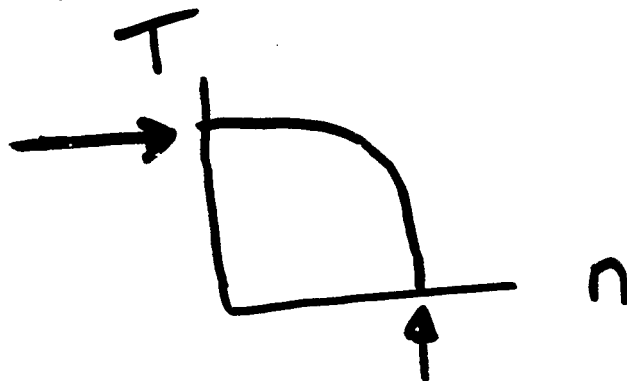
PHASE TRANSITION

Physical picture suggested by PGO has implications for phase transition to a chiral symmetry restored phase:

Percolation of pre-existing bags of symmetric phase.

Very different from finite T transition.

Condensate melts



(no contradiction, since $T=T_c$, $n=0$ has mostly pions, few baryons)

Condensate gets eaten away by more and more holes.

Color Superconductivity at high density

M. Alford (IAS)

K. Rajagopal (MIT)

F. Wilczek (IAS)

I The Lorentz scalar colored condensate

Gap, equation of state, gauge symmetries

II The axial vector colored condensate

exotic but fragile

III Future Directions

At high enough density, attractive quark-quark interactions in QCD should cause condensation of Cooper pairs (diquarks).

In our model, the instanton vertex provides an attractive interaction, and a colored diquark condensate forms.

$$1. \quad \langle u^\alpha C \gamma_5 d^\beta \rangle \varepsilon_{\underline{\alpha}\underline{\beta}3}, \quad \text{Lorentz scalar, color } \bar{3}.$$

Robust.

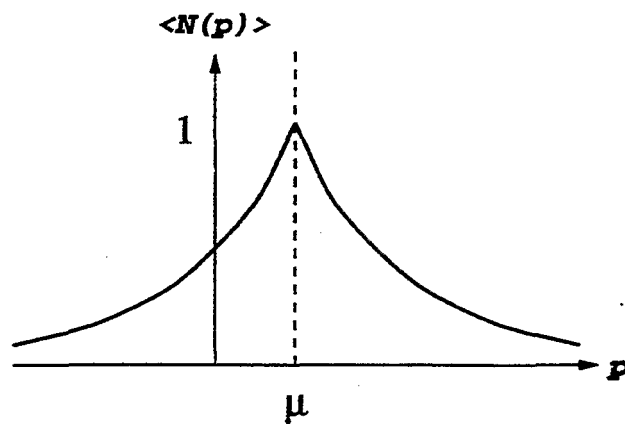
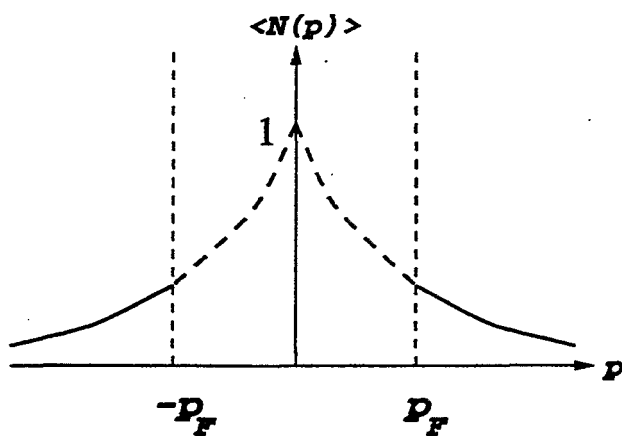
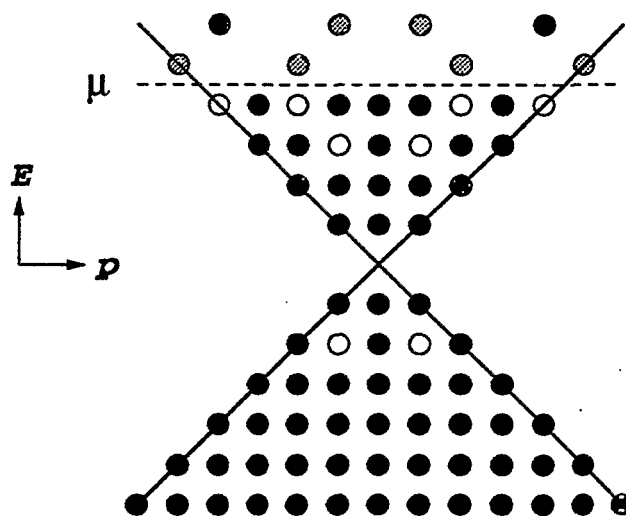
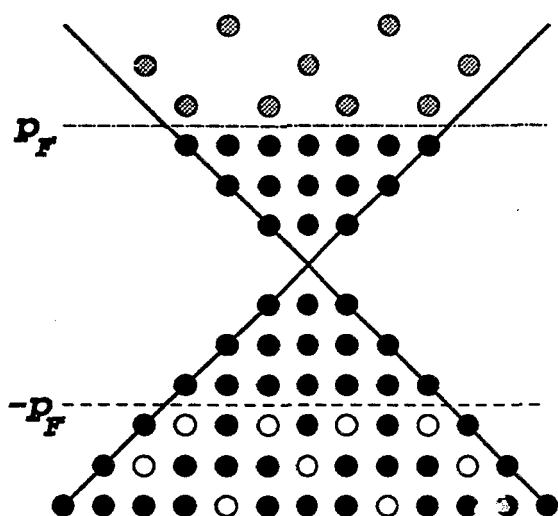
Gap ~ 100 MeV

$$2. \quad \langle u^{\underline{3}} C \sigma^{0z} d^{\underline{3}} \rangle, \quad \text{Lorentz axial vector, color } 6.$$

Sensitive.

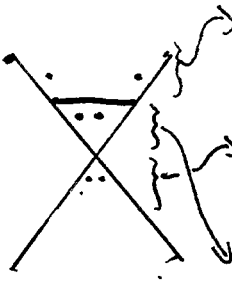
Gap ~ 1 keV

Comparing Chiral and Colored condensates



I. Lorentz scalar colored condensate

The ansatz:



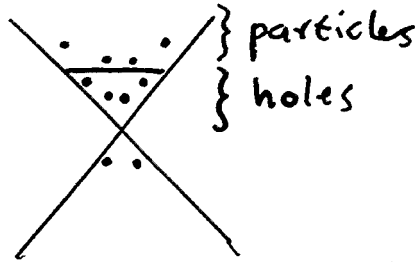
$$\begin{aligned}
 |\psi\rangle = & \prod_{\alpha, \beta, \mathbf{p}} \left(\cos(\theta_{A\mathbf{p}}^L) + \varepsilon^{\underline{\alpha}\underline{\beta}3} \sin(\theta_{A\mathbf{p}}^L) e^{i\xi_A^L(\mathbf{p})} a_{L1\underline{\alpha}}^\dagger(\mathbf{p}) a_{L2\underline{\beta}}^\dagger(-\mathbf{p}) \right) \\
 & \left(\cos(\theta_{B\mathbf{p}}^R) + \varepsilon^{\underline{\alpha}\underline{\beta}3} \sin(\theta_{B\mathbf{p}}^R) e^{i\xi_B^R(\mathbf{p})} b_{R1\underline{\alpha}}^\dagger(\mathbf{p}) b_{R2\underline{\beta}}^\dagger(-\mathbf{p}) \right) \\
 & \left(\cos(\theta_{C\mathbf{p}}^R) + \varepsilon^{\underline{\alpha}\underline{\beta}3} \sin(\theta_{C\mathbf{p}}^R) e^{i\xi_C^R(\mathbf{p})} c_{R1\underline{\alpha}}^\dagger(\mathbf{p}) c_{R2\underline{\beta}}^\dagger(-\mathbf{p}) \right) \\
 & \times (L \leftrightarrow R) \quad \times \quad |F(p_F)\rangle
 \end{aligned}$$

$|F(p_F)\rangle$ is the Fermi sea filled up to momentum p_F .

This state is antisymmetric in color, flavor, and spin.

It selects the color 3 direction.

Minimizing $E - \mu N$ with respect to the $\theta(\mathbf{p})$, we get the gap equation.

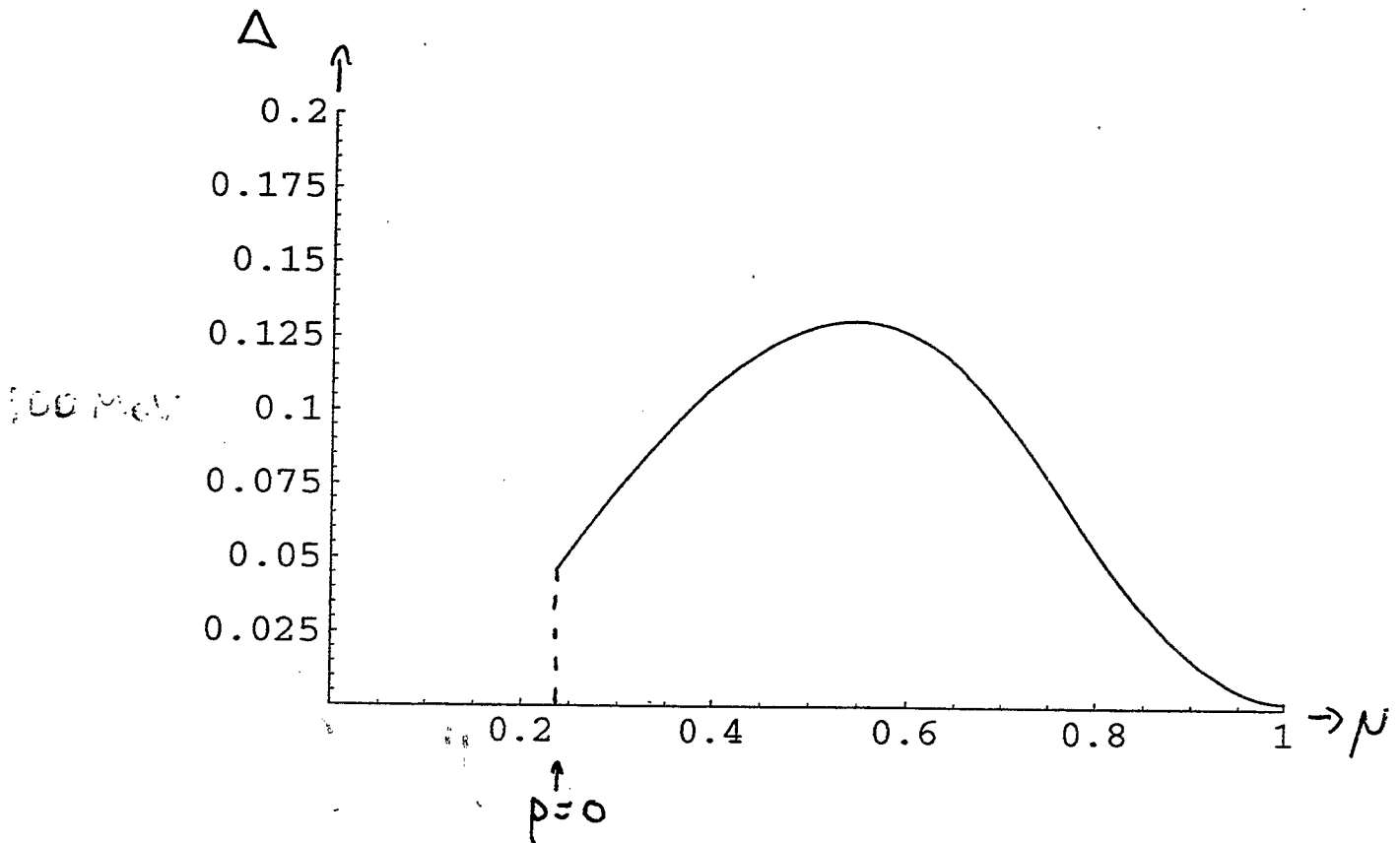


The gap equation is

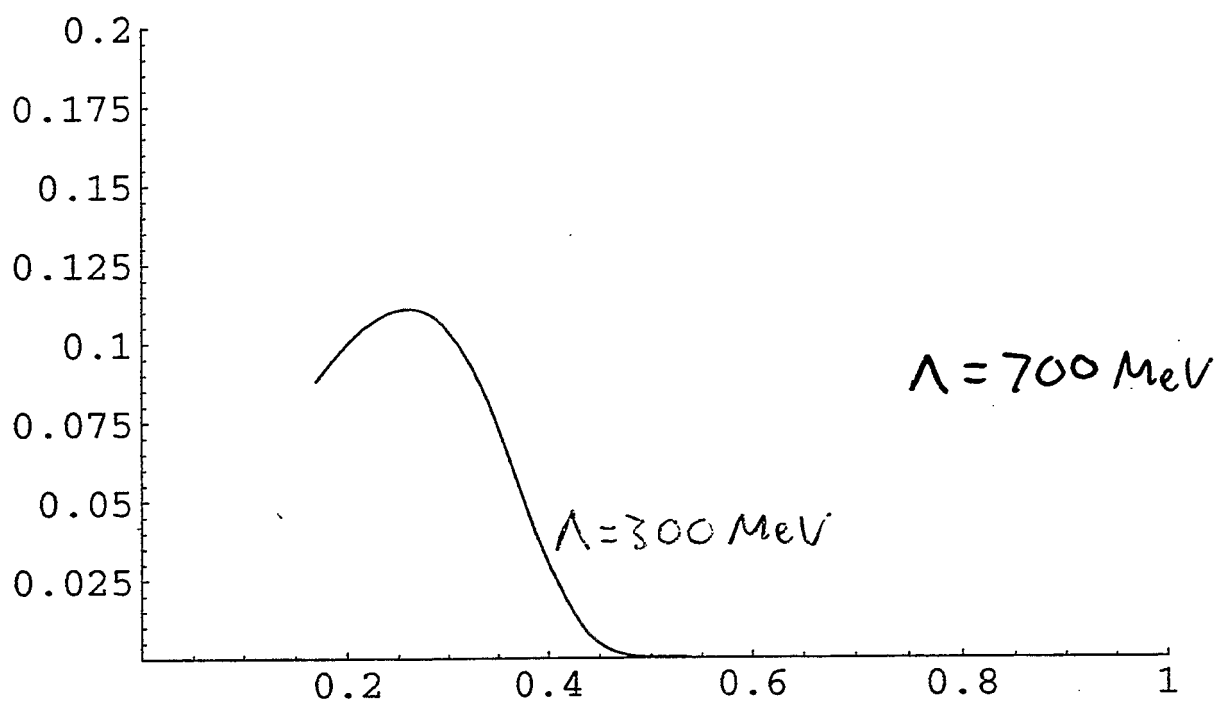
$$1 = \frac{2K}{V} \left\{ \begin{aligned} &\sum_{p > \mu} \frac{f_{\Lambda}(p)}{\sqrt{\Delta^2 + (p - \mu)^2 / f_{\Lambda}^2(k)}} \quad \left. \vphantom{\sum_{p > \mu}} \right\} \text{particles} \\ &+ \sum_{p < \mu} \frac{f_{\Lambda}(p)}{\sqrt{\Delta^2 + (\mu - p)^2 / f_{\Lambda}^2(k)}} \quad \left. \vphantom{\sum_{p < \mu}} \right\} \text{holes} \\ &+ \sum_p \frac{f_{\Lambda}(p)}{\sqrt{\Delta^2 + (p + \mu)^2 / f_{\Lambda}^2(k)}} \quad \left. \vphantom{\sum_p} \right\} \end{aligned} \right.$$

Note BCS divergence as $\Delta \rightarrow 0$: there is *always* a solution, for any interaction strength K and chemical potential μ .

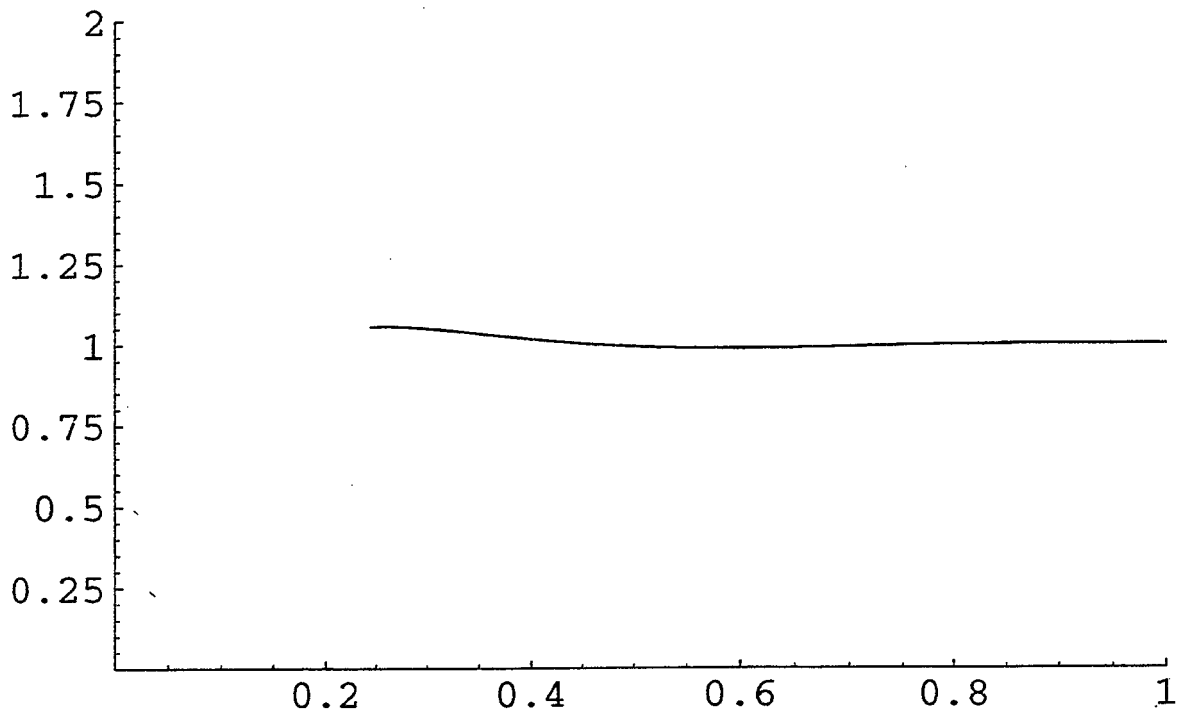
Color gap vs chemical potential (GeV), $\Lambda = 700$ MeV.



- Maximum gap is ~ 100 MeV, smaller than for chiral at $\mu = 0$ but same order of magnitude.
- It is suppressed by phase space (small Fermi surface) as $\mu \rightarrow 0$, and by asymptotic freedom (decoupling of the vertex) at large μ .
- Robust vs changes in Λ



Density with SC condensate compared to free quarks,
 $\Lambda = 700 \text{ MeV}$.



The superconducting condensate has little effect on the equation of state.

Gauge symmetries

Long-range interactions are affected by the condensate.

Instead of color and electromagnetism (9 massless gauge bosons) we have a reduced color group and a rotated electromagnetism (4 massless gauge bosons)

$$SU(3)_c \times U(1)_Q \rightarrow SU(2)_c \times U(1)_{Q'}$$

$$Q = \frac{1}{2}(B + I_3) \qquad Q' = Q + \frac{1}{6}T_8$$

quark	Q' charge	} condensate
$u^{\text{red,green}}$	$+\frac{1}{2}$	
$d^{\text{red,green}}$	$-\frac{1}{2}$	
u^{blue}	1	
d^{blue}	0	

Axial condensate

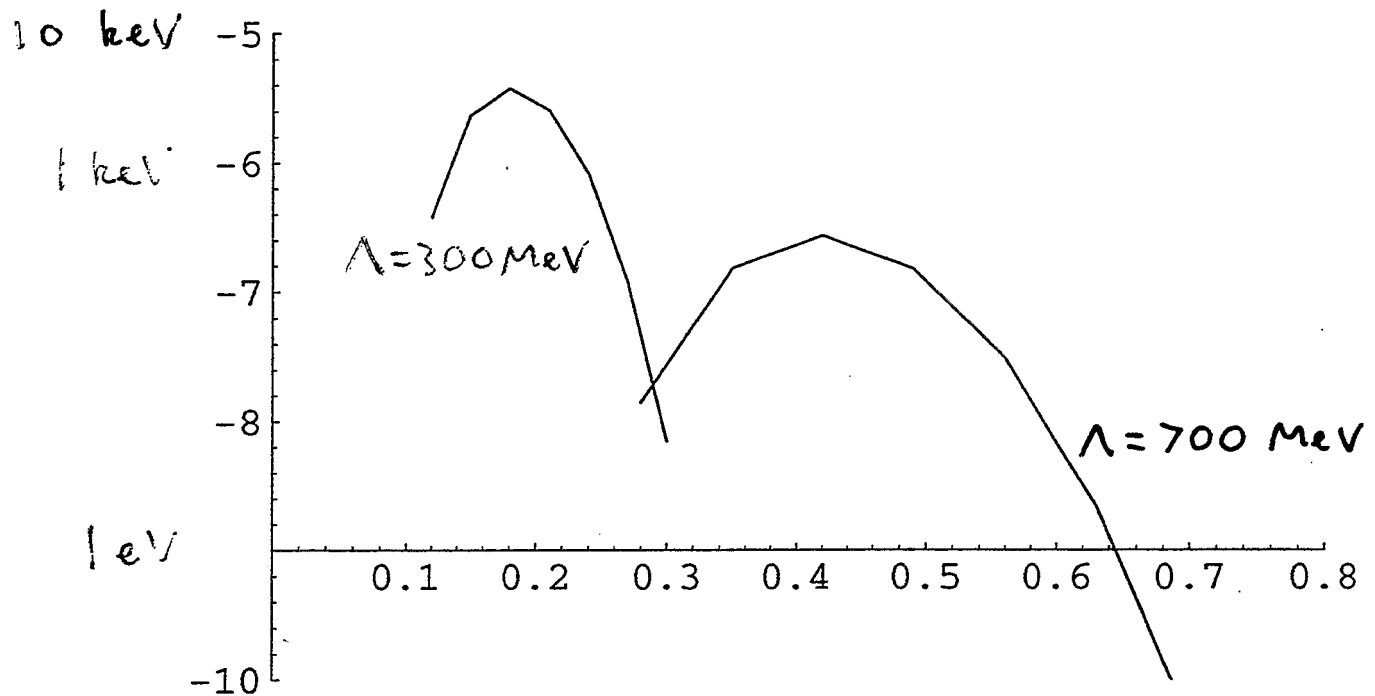
The ansatz:

$$\begin{aligned}
 |\psi\rangle = & \prod_{\mathbf{p}} \frac{1}{\sqrt{1+(\tau_A^L(p)p_z/p)^2}} \left(1 + \tau_A^L(p) \frac{p_z}{p} a_{L\,1\underline{3}}^\dagger(\mathbf{p}) a_{L\,2\underline{3}}^\dagger(-\mathbf{p}) \right) \\
 & \frac{1}{\sqrt{1+(\tau_B^R(p)p_z/p)^2}} \left(1 + \tau_B^R(p) \frac{p_z}{p} b_{R\,1\underline{3}}^\dagger(\mathbf{p}) b_{R\,2\underline{3}}^\dagger(-\mathbf{p}) \right) \\
 & \frac{1}{\sqrt{1+(\tau_C^R(p)p_z/p)^2}} \left(1 + \tau_C^R(p) \frac{p_z}{p} c_{R\,1\underline{3}}^\dagger(\mathbf{p}) c_{R\,2\underline{3}}^\dagger(-\mathbf{p}) \right) \\
 & \times (L \leftrightarrow R) \quad \times \quad |F(p_F)\rangle
 \end{aligned} \tag{1}$$

This state is $(6_S, 1_A)$ in color and flavor, 1_S^+ in spin. It selects the color $\underline{3}$ direction and the spatial \underline{z} direction.

Minimizing $E - \mu N$ with respect to the $\tau(p)$ gives us a gap equation.

log(Axial vector gap) vs chemical potential (GeV).



As before, the BCS singularity at the Fermi surface guarantees a solution, but only one color participates, and not all p fully participate, so the gap turns out to be very small and sensitive to the cutoff.

No robust prediction is possible. Other interactions may push it up or down.

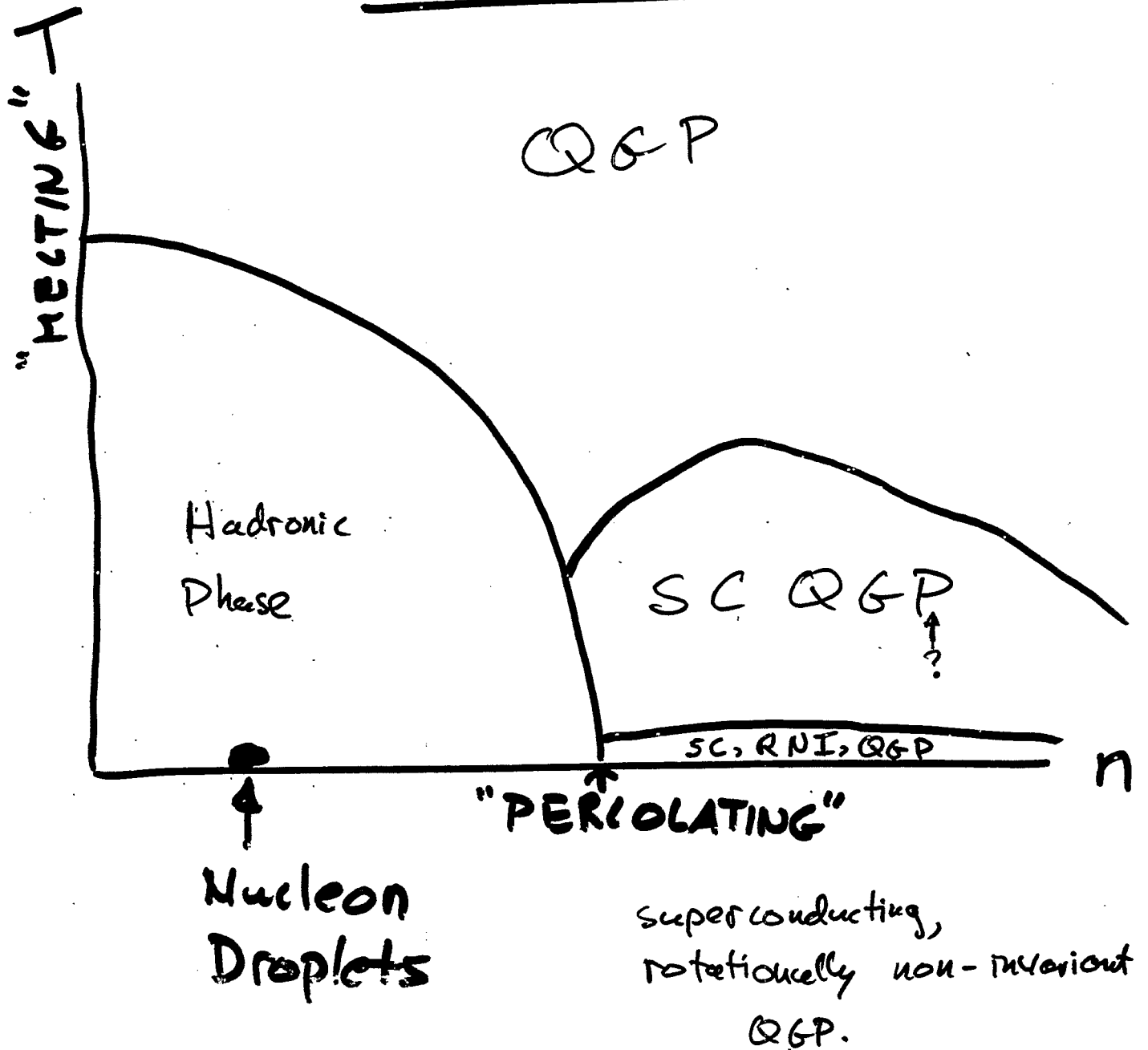
Summary of Condensates

	Chiral	SC	Ax
	$\langle \bar{q}q \rangle$	$\langle u^\alpha C \gamma_5 d^\beta \rangle \varepsilon_{\alpha\beta 3}$	$\langle u^3 C \sigma^{0z} d^3 \rangle$
$SU(3)_c$	1	$\bar{3}$	6
$SU(2)_V$	1	1	1
$SU(2)_A$	3	1	1
$U(1)_B$	1	$e^{i\pi/3}$	$e^{i\pi/3}$
Spin^P	0^+	0^+	1^+
Gauge sym	$SU(3)_c$ $\times U(1)_Q$ $Q = \frac{1}{2}(B + I_3)$	$SU(2)_c$ $\times U(1)_{Q'}$ $Q' = T_8 + \alpha Q$	$SU(2)_c$
Gauge bosons	$8 + 1 = 9$	$3 + 1 = 4$	3
Massive G.B.	0	5	6

Conclusions

- High-density state is not a Coulomb phase of free quarks and gluons, but is broken down to $SU(2) \left[\times U(1)_{Q'} \right]$. This can be described as a Higgs or a confined phase:
Confinement without chiral symmetry breaking.
- We have not yet found a signature of these condensates for heavy ion collisions. With a bigger gap, axial vector condensate would give striking polarization correlations.
- Neutron stars may be sensitive to condensates. E.g., a gap will slow down cooling by neutrino emission.

AS PROMISED, THE CARTOON REDRAWN



- Finite temperature. What is $T_c(\mu)$? (Expect $\sim \Delta$)
- Strange quark: Six leg instanton vertex
 - new possibilities, eg $\langle q_{i\alpha} q_{j\beta} \rangle \epsilon^{ij\mathbf{B}} \epsilon^{\alpha\beta\mathbf{B}}$ breaking to diagonal of color and flavor.
 - explore m_s dependence.
 - condensate will have to break flavor as well as color, hence local order parameter.
- Other interactions: gluons, electromagnetism, effective interactions.
- Other condensates
- Details of non-uniform chiral breaking phase: droplet size, percolation-driven restoration of chiral symmetry.
- Consequences for neutron stars (cooling...)
- Consequences for heavy ion collisions (screening of broken gauge fields in quark-gluon plasma?).

Instantons and Color Superconductivity

T. Schäfer

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98195

Abstract

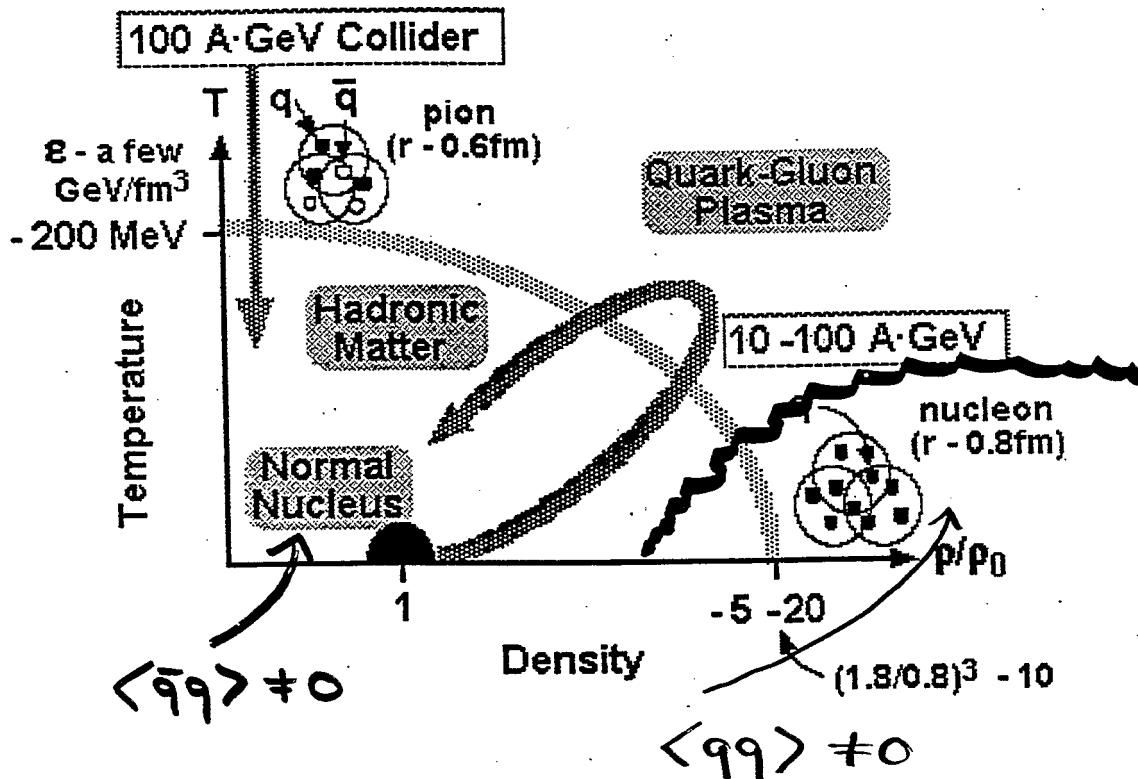
Instantons lead to strong correlations between up and down quarks with spin zero and anti-symmetric color wave functions. In cold and dense matter, $n_b > n_c \simeq 1 fm^{-3}$ and $T < T_c \sim 50 MeV$, these pairs Bose-condense, replacing the usual $\langle \bar{q}q \rangle$ condensate and restoring chiral symmetry. At high density, the ground state is a color superconductor in which diquarks play the role of Cooper pairs. An interesting toy model is provided by QCD with two colors: it has a particle-anti-particle symmetry which relates $\langle \bar{q}q \rangle$ and $\langle qq \rangle$ condensates.

INSTANTONS AND COLOR

SUPER CONDUCTIVITY

T. SCHAEFER, INT SEATTLE

Phase Diagram of Nuclear Matter & Nuclear Collisions



COLOR SUPERCONDUCTIVITY

- $T=\mu=0$: STRONG $\bar{q}^a q^a$ INTERACTION LEADS TO CONDENSATION OF $J^P=0^+$ PAIRS
ALSO: LARGE CORRELATION IN 0^+ COLOR $\bar{3}$ $q\bar{q}$ CHANNEL
- $\mu > 0$: $\langle \bar{q}q \rangle$ DROPS \leadsto X SCH. REST.
- COOPER PHENOMENON: ARB. WEAK, ATTRACTIVE INTERACTION NEAR FERMI SURFACE LEADS TO PAIR CONDENSATION
- LARGEST GAP: $\sum^{abc} q^b C_{\bar{5}5} q^c$ ($J^P=0^+$, $u\downarrow d\uparrow$)
- INSTANTON EFFECTS \leadsto LARGE GAP $O(100 \mu V)$
BAILIN & LOVE (1984) $\Delta \approx (1-10) \mu V$

MODEL SYSTEM: $N_c=2$

- EXTRA SYMMETRY: $\bar{q}^a q^a \rightarrow \sum^{ab} q^a C_{\bar{5}5} q^b$
 $\mu \neq 0 \leadsto \langle \bar{q}q \rangle \neq 0$ $\mu > \mu_c$ $\langle q C_{\bar{5}5} q \rangle \neq 0$
- NO SIGN PROBLEM \leadsto SIMULATIONS FEASIBLE

INSTANTON LIQUID AT $T=0$

- QCD PARTITION FUNCTION

$$Z = \int DA_\mu e^{-S} \det(D + m)$$

$$S = \frac{1}{4g^2} \int d^4x Q^2$$

- ASSUME: DOMINATED BY CLASSICAL CONFIGURATIONS \leadsto "INSTANTONS"

$$\int DA_\mu \rightarrow \sum_{N_f, N_+} \frac{1}{N_+! N_-!} \int \prod_i^{N_+} (d\rho_i d^4z_i d\mu_i)$$

$$A_\mu^a = \frac{2}{g} \Omega^{ab} \gamma_{\mu 0}^b \frac{(x-z)_0}{(x-z)^2 + \rho^2}$$

COLOR \rightarrow
 \leftarrow POSITION
SIZE \leftarrow

- INSTANTON $\hat{=}$ TUNNELING EVENT

$$\text{RATE} \sim e^{-S}$$



- TUNNELING LOWERS GROUND STATE

$$\mathcal{E} \simeq -\frac{b}{4} \left(\frac{N}{V} \right) \leftarrow \text{TRACE ANOMALY}$$

$$\mathcal{E} \simeq -\frac{b}{32} \left\langle \frac{\alpha}{\pi} Q^2 \right\rangle$$

$$\mathcal{E} \simeq -500 \text{ MeV/fm}^3 \leadsto \left(\frac{N}{V} \right) \simeq 1 \text{ fm}^{-4}$$

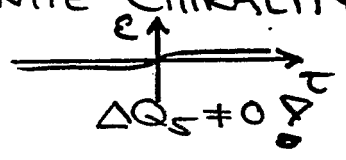
(THE MOST IMPORTANT NUMBER!)

QUARKS IN THE INSTANTON VACUUM

- ONE INSTANTON: DIRAC OPERATOR HAS ZERO MODE

$$i\mathcal{D}\psi_0^{I,A} = 0 \quad \gamma_5 \psi_0^{I,A} = \pm \psi_0^{I,A} \rightarrow \text{DEFINITE CHIRALITY}$$

- MANY INSTANTONS: ZERO MODES INTERACT

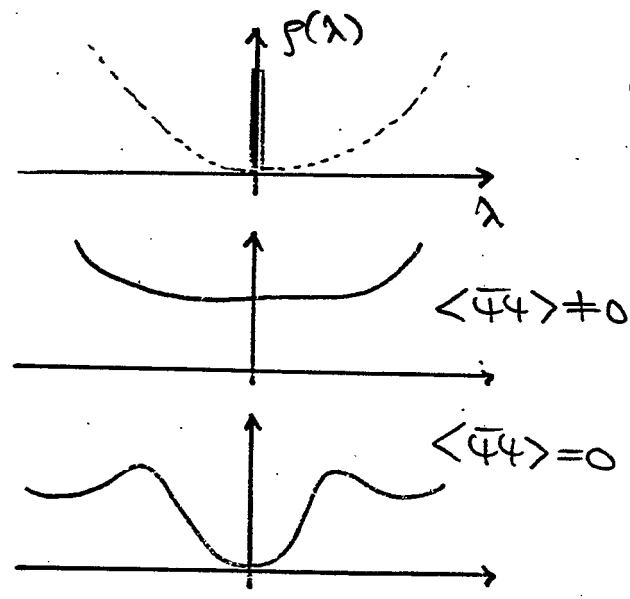


$$i\mathcal{D} = \begin{pmatrix} 0 & T_{IA} \\ T_{IA}^\dagger & 0 \end{pmatrix} \quad T_{IA} = \int d^4x \psi_I^\dagger i\mathcal{D} \psi_A$$

- QUARK CONDENSATE (CASHER-BANKS)

$$\langle \bar{q}q \rangle = \frac{1}{\pi} \rho(\lambda=0)$$

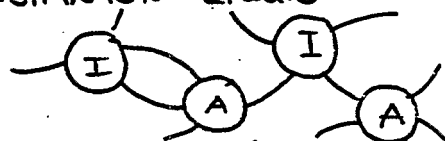
COMPARE:
 $G = e^2 D\rho(E_F)$
 (WONDO)



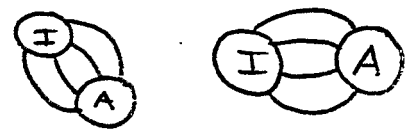
ONE INSTANTON



INSTANTON LIQUID



INSTANTON MOLECULES



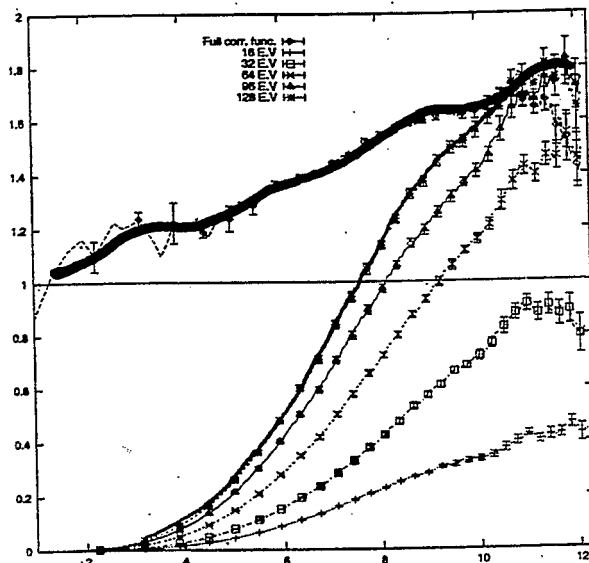
WHY
INSTANTONS?

DOMINATE DIRAC SPECTRUM
AT SMALL VIRTUALITY

IMPORTANCE OF LOW LYING MODES

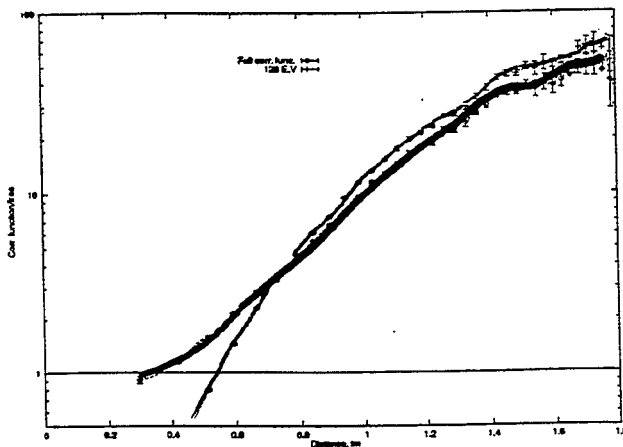
IVANENKO, NEGELE

$$\pi_f / \pi_f^{(0)}$$



— Full
— 128 MODES

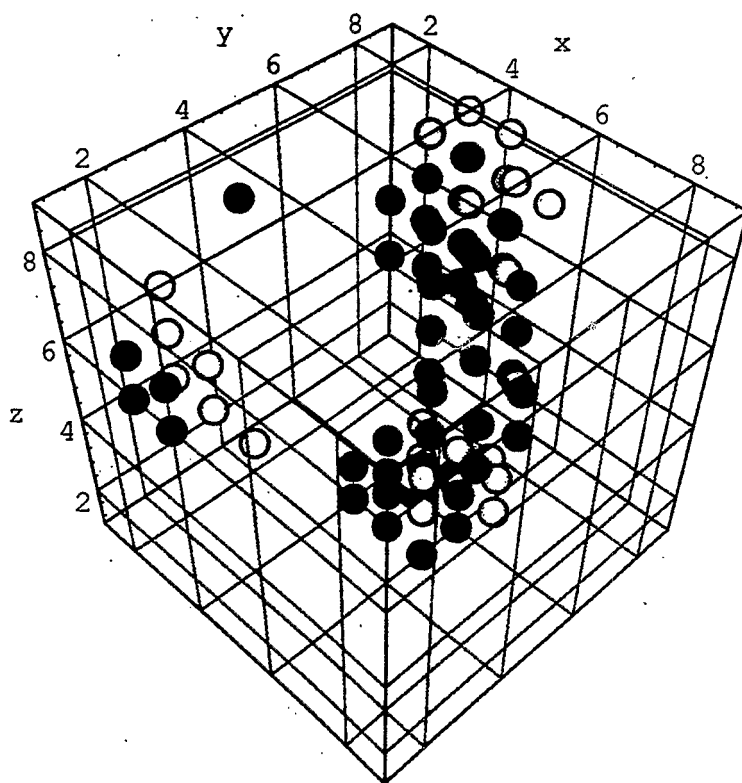
$$\pi_\pi / \pi_\pi^{(0)}$$



$$iD\psi_\lambda = \lambda\psi_\lambda$$

$$\leadsto S(x,y) = \sum_\lambda \frac{\psi_\lambda(x) \psi_\lambda^\dagger(y)}{\lambda + i\epsilon}$$

CORRELATION OF INSTANTONS & CHIRAL SYMMETRY BREAKING



- TOPOLOGICAL CHARGES ± 1
- CHIRAL CONDENSATE

MARKUM ET AL, PREPRINT (97)

CORRELATORS IN THE INSTANTON LIQUID

• MESONIC CORRELATORS

ENSEMBLE AVERAGE

$$\Pi_\Gamma(x) = \langle \text{tr} (\Gamma S(x) \Gamma S(-x)) \rangle \quad \Gamma = \gamma_5, \dots$$

• PROPAGATOR IN THE INSTANTON FIELD

$$S(x) = \frac{\phi_0^1(x) \phi_0^{\dagger}(0)}{i\omega} + S_{\text{NEW}}(x)$$

$$\phi_0(x) = \frac{\rho}{\pi} \frac{1}{(x^2 + \rho^2)^{3/2}} \gamma \cdot \hat{x} \frac{1 + \gamma_5}{2} \chi$$

$\sim 1/x^3$ DET. SPIN-SPIN INTERACTION

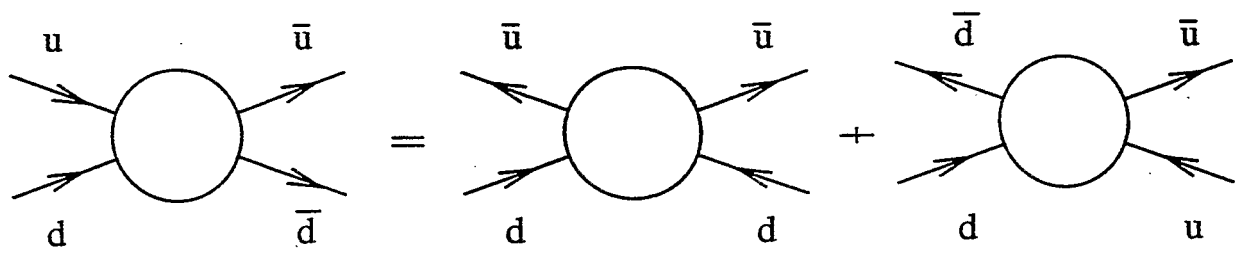
• AMPUTATE EXTERNAL LEGS \rightarrow EFF $(2N_f)$ QUARK INT.

$$\mathcal{L} = \underbrace{q \varphi^4(k)}_{\text{ZERO MODE FORM FACTOR}} \left\{ \frac{1}{N_c^2} [(\bar{\psi} \tau_a \psi)^2 + (\bar{\psi} \gamma_5 \tau_a \psi)^2] \right\}$$

ZERO MODE FORM FACTOR

$$(\tau_a)(\tau_a) = (\vec{\tau})(\vec{\tau}) - (1)(1)$$

DIRECT + EXCHANGE TERMS



- ATTRACTIVE IN $(5, \pi)$, REP. IN $(3, 8)$ CHANNEL
- RPA ($\hat{=}$ LEADING ORDER DIAGRAM IN (N_f^4))

$$\Pi_\pi(Q^2) = \Gamma_S(Q^2) \frac{1}{1 - C_S(Q^2)} \Gamma_S(Q^2)$$

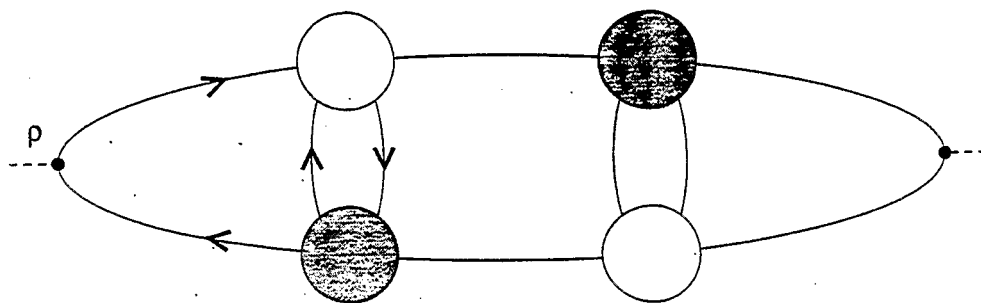
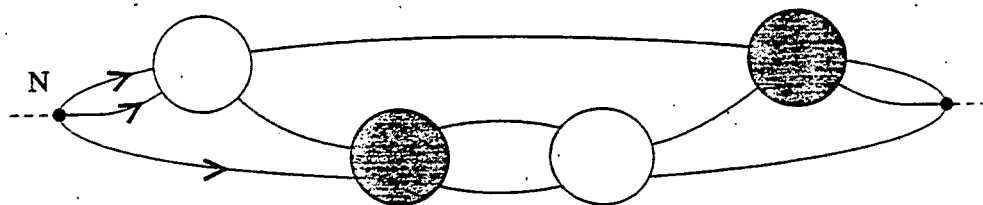


- ALL ORDERS: INTERACTING INSTANTON LIQUID

$$S(x) = \sum_{I,J} \phi_I(x) \left(\frac{1}{D + m} \right)_{IJ} \phi_J^+(0) + \dots$$

- DIAGONALIZE (D) IN ZERO MODE BASIS
 \rightarrow COLLECTIVE EIGENSTATES

- CORRELATORS $\Pi_F(x) = \langle \text{tr} (S(x) \Gamma S(-x) \Gamma) \rangle$



PHENOMENOLOGY OF EFF. INTERACTION

- QQ INTERACTION NOT ACCESSIBLE FROM EXPERIMENT
 \rightarrow STUDY HADRONIC CORRELATORS

$$\Pi_T(x) = \langle j_T(x) j_T(0) \rangle$$

IN THE WINDOW $\Lambda_{\text{KSB}}^{-1} < x < \Lambda_{\text{CONF}}^{-1}$

$$\Lambda_{\text{KSB}}^{-1} \simeq 0.2 \text{ fm} \quad \Lambda_{\text{CONF}}^{-1} \simeq 1.0 \text{ fm}$$

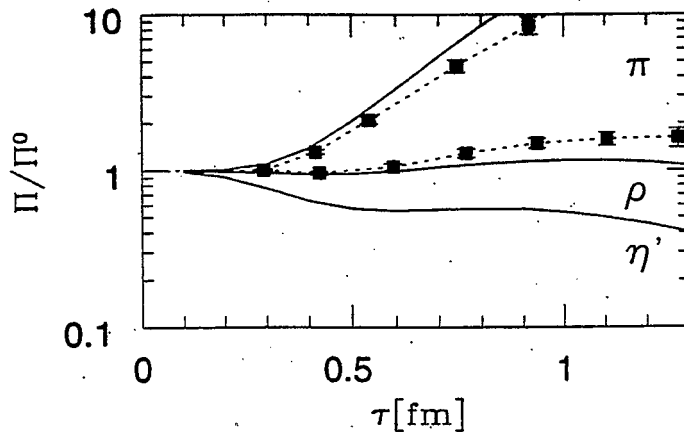
- RESULT CAN BE REPRESENTED IN TERMS OF EFF. LAGRANGIAN

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + \sum_r \frac{c_r}{\Lambda^2} (\bar{\psi}\Gamma\psi)^2 + \sum_r \frac{d_r}{\Lambda^5} (\bar{\psi}\Gamma\psi)^3 + \dots$$

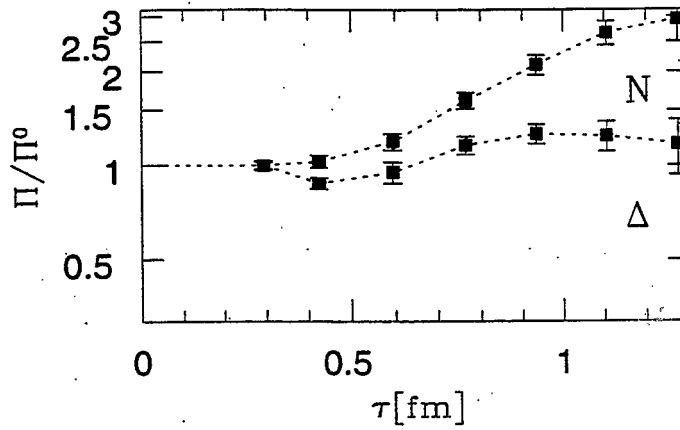
- INSTANTONS PROVIDE

- SPIN-ISOSPIN STRUCTURE
- KSB - SCALE Λ
- EXPANSION PARAMETER ($p^4 N_c$)

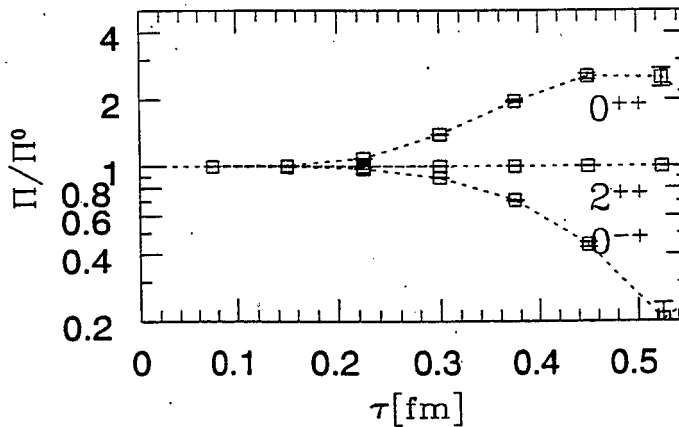
HADRONIC CORRELATION FUNCTIONS



MESONS



BARYONS



GLUEBALLS

- EXPERIMENTAL SPEC. FCT.
- LATTICE DATA (CHU ET AL.)
- INSTANTON CALCULATION

DIQUARK CURRENTS

• STRUCTURE: $J = q C \Gamma_{\text{DIRAC}} \Gamma_{\text{COLOR}} \Gamma_{\text{FLAVOR}} q$

• $\Gamma_1 \Gamma_2 \Gamma_3$ HAS TO BE ANTI-SYMMETRIC

$$\begin{aligned} \Gamma_D &= \{ C \gamma_5, C, C \gamma_\mu \gamma_5 \} & \text{ASYM} \\ \Gamma_D &= \{ C \gamma_\mu, C \gamma_{\mu\nu} \} & \text{SYM} \end{aligned}$$

$$\begin{aligned} \Gamma_C &= \lambda_A = \{ \lambda_2, \lambda_5, \lambda_7 \} & \text{COLOR } \bar{3}, \text{ ASYM} \\ \Gamma_C &= \lambda_8 = \{ \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8 \} & 6, \text{ SYM} \end{aligned}$$

$$\begin{aligned} \Gamma_F &= \tau_2 \begin{pmatrix} \lambda_A^F \\ \lambda_S^F \end{pmatrix} & I=0, \text{ ASYM} \\ \Gamma_F &= \tau_2 \hat{\tau} \begin{pmatrix} \lambda_A^F \\ \lambda_S^F \end{pmatrix} & I=1, \text{ SYM} \end{aligned}$$

• PICK SPIN-COLOR \Rightarrow FLAVOR FIXED

$$J^a = q C \gamma_5 \lambda_A^a \tau_2 q = \epsilon^{abc} q^b C \gamma_5 \tau_2 q^c \quad J^\pi = 0^+$$

• PHYSICS: NOT GAUGE INVARIANT, BUT IMPORTANT BUILDING BLOCK FOR BARYON CURRENTS

$$J_P = J_S^a u^a = \epsilon^{abc} (u^b C \gamma_5 d^c) u^a$$

$$J_A = J_\mu^a (\gamma_\mu u^a) = \epsilon^{abc} (u^b C \gamma_\mu \gamma_5 u^c) \gamma_\mu u^a$$

- HEAVY-LIGHT SYSTEMS

$$\Lambda_Q: (u\bar{c}\bar{s}d)Q, (u\bar{c}d)\gamma_5 Q, (u\bar{c}\mu\bar{s}d)\gamma_\mu Q$$

$$\Sigma_Q: (u\bar{c}\mu d)\gamma_\mu\gamma_5 Q, (u\bar{c}\mu d)\gamma_5\gamma_\mu Q$$

- HEAVY QUARK LIMIT, $M_Q \rightarrow \infty$

$$\bar{Q}(x)Q(0) \rightarrow P \exp(i \int A_\mu dx_\mu)$$

$$\langle \bar{\Lambda}_Q(x) \Lambda_Q(0) \rangle = \langle \int \bar{s}(x) P(x,0) s(0) \rangle$$

\Rightarrow QI DIQ CORRELATOR

- PHENOMENOLOGY

$$m_P < m_\Delta, \quad m_{\Lambda_Q} < m_{\Sigma_Q} \Rightarrow m_S < m_V$$

- ALSO: $\langle 0 | j_N^{12} | N \rangle = \lambda_2 u \quad j^1 = \bar{s}^a \cdot u^a, \quad j^2 = \bar{s}^a \gamma_5 u^a$

$$\lambda_1 \gg \lambda_2 \Rightarrow \text{STRONG CORRELATIONS IN SCAL. DIQ CHANNEL}$$

EFF. DIQUARK INTERACTION

- FIERZ REARRANGE $Q\bar{Q}$ INTERACTION

$$\mathcal{L} = G \psi^4(k) \left\{ \frac{1}{2N_c(N_c-1)} \left[(4C\gamma_5\tau_2\lambda^A\psi)(\bar{\psi}C\gamma_5\tau_2\lambda^A\bar{\psi}) \right. \right. \\ \left. \left. + (4C\tau_2\lambda^A\psi)(\bar{\psi}C\tau_2\lambda^A\bar{\psi}) \right] \right. \\ \left. + \frac{1}{4N_c(N_c+1)} (4C\sigma_{\mu\nu}\tau_2\lambda^3\psi)(\bar{\psi}C\sigma_{\mu\nu}\tau_2\lambda^3\bar{\psi}) \right\}$$

↳ ATTRACTIVE FOR SCALAR ($\bar{3}$) DIQUARKS
 REPULSIVE IN PSEUDO-SCALAR CHANNEL

↳ TENSOR COUPLES TO VECTOR/AXIAL (6) STATES
 $(4C\sigma_{ij}\tau_2\lambda^3\psi) / (4C\sigma_{ij}\tau_2\lambda^8\psi)$

- SIMULATIONS

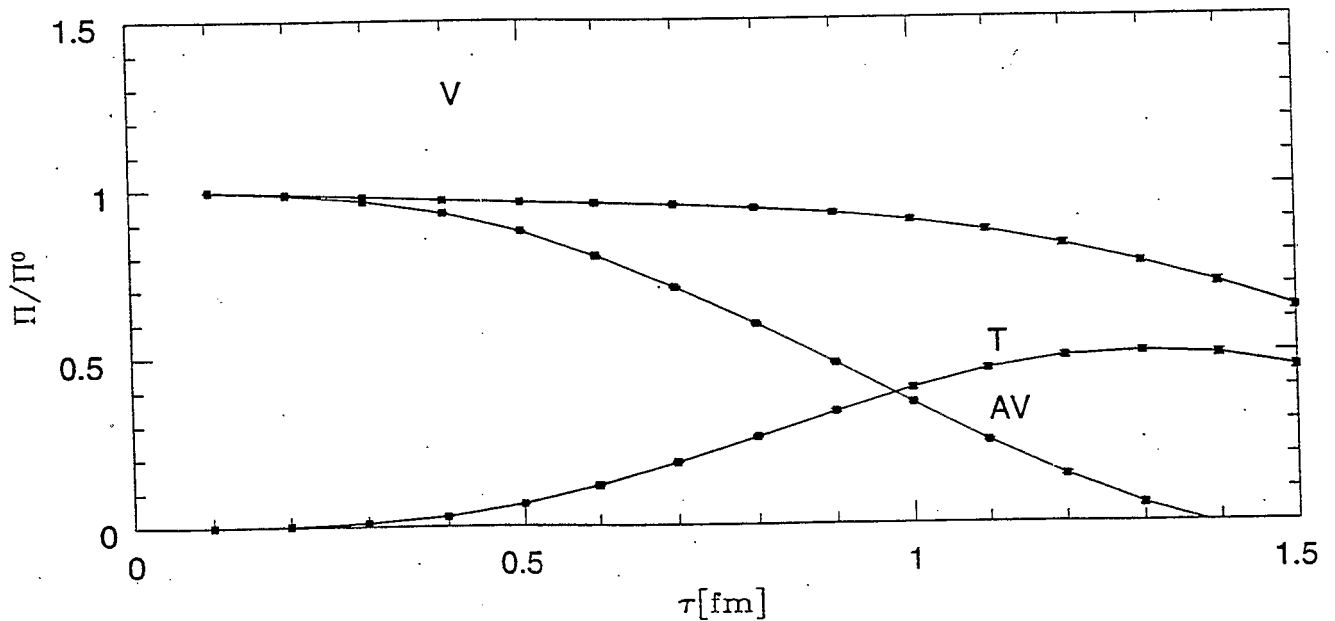
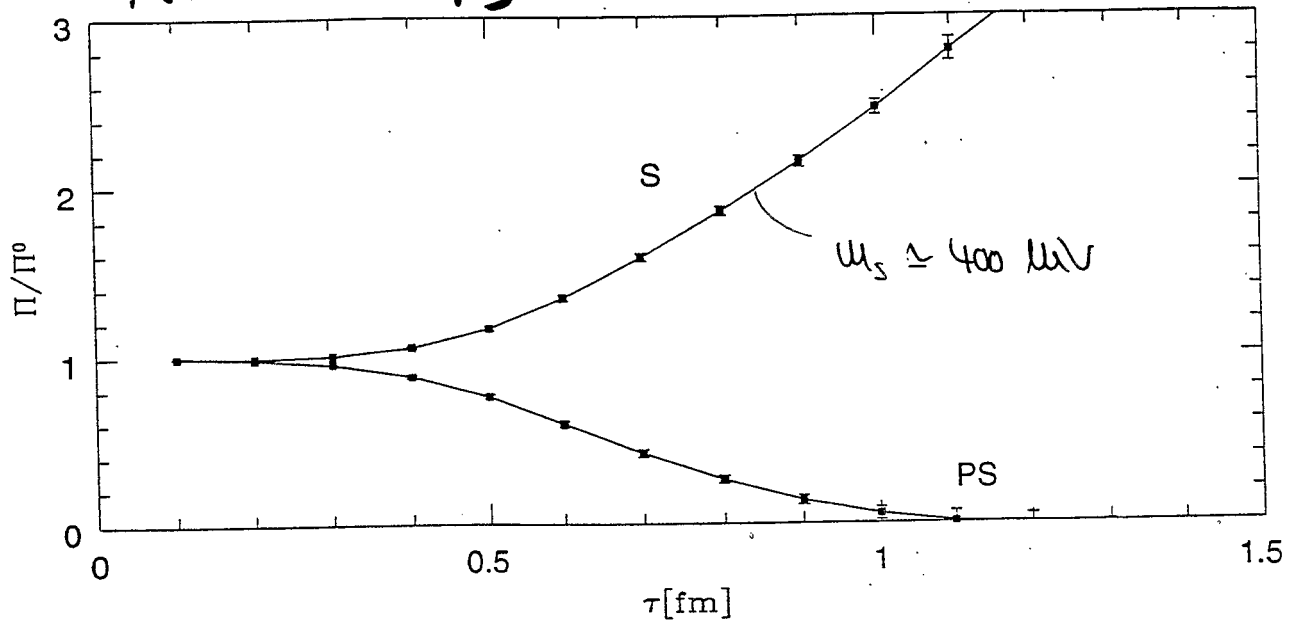
$$m_S \simeq 400 \text{ MeV}$$

$$\rightarrow 2m_q - m_S \simeq (200-300) \text{ MeV}$$

$$m_S^{(us)} \simeq$$

DIQUARK CORRELATION FUNCTIONS

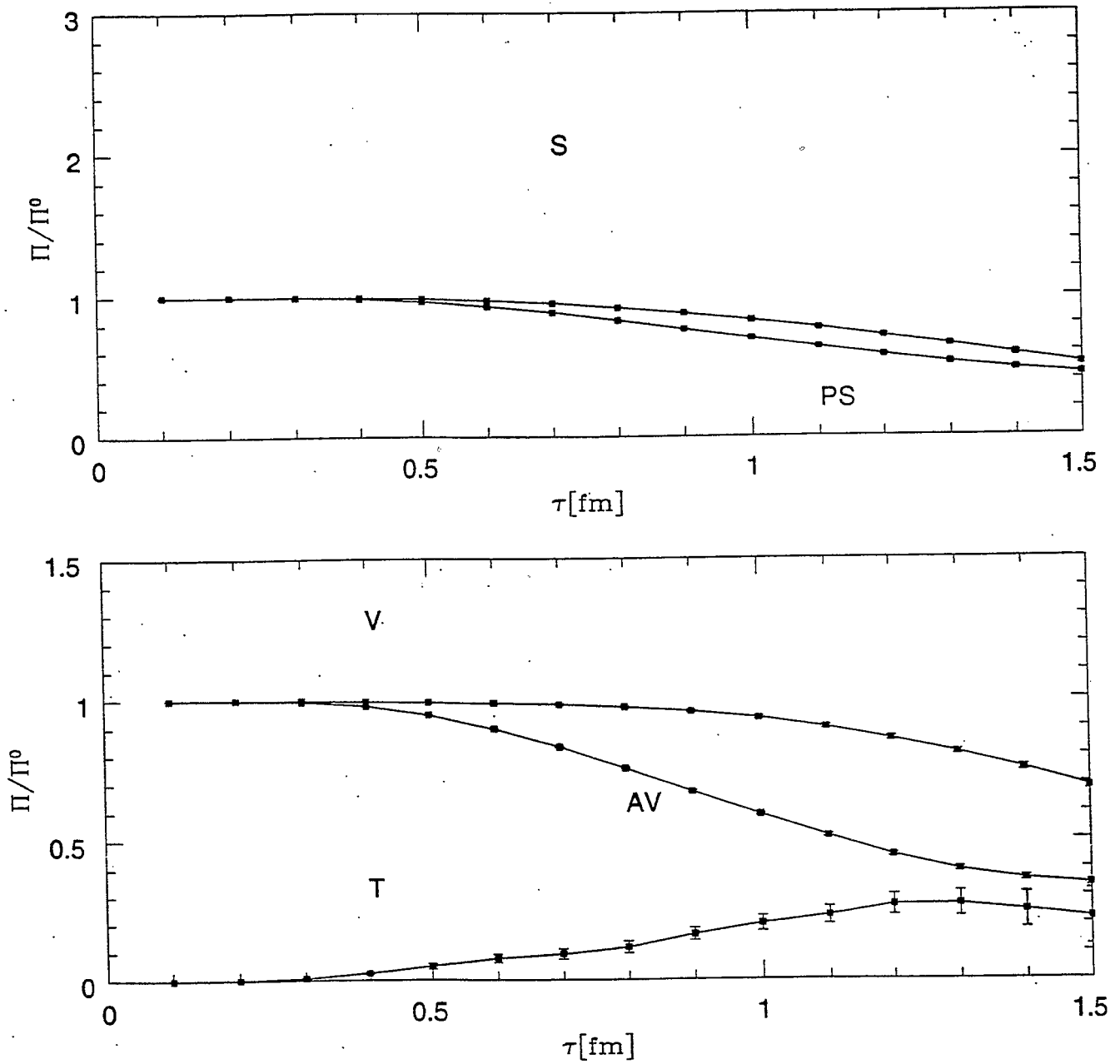
NON-STRANGE, COLOR $\bar{3}$



$$j_r^a = \epsilon^{abc} q^b \tau_r^c q^c$$

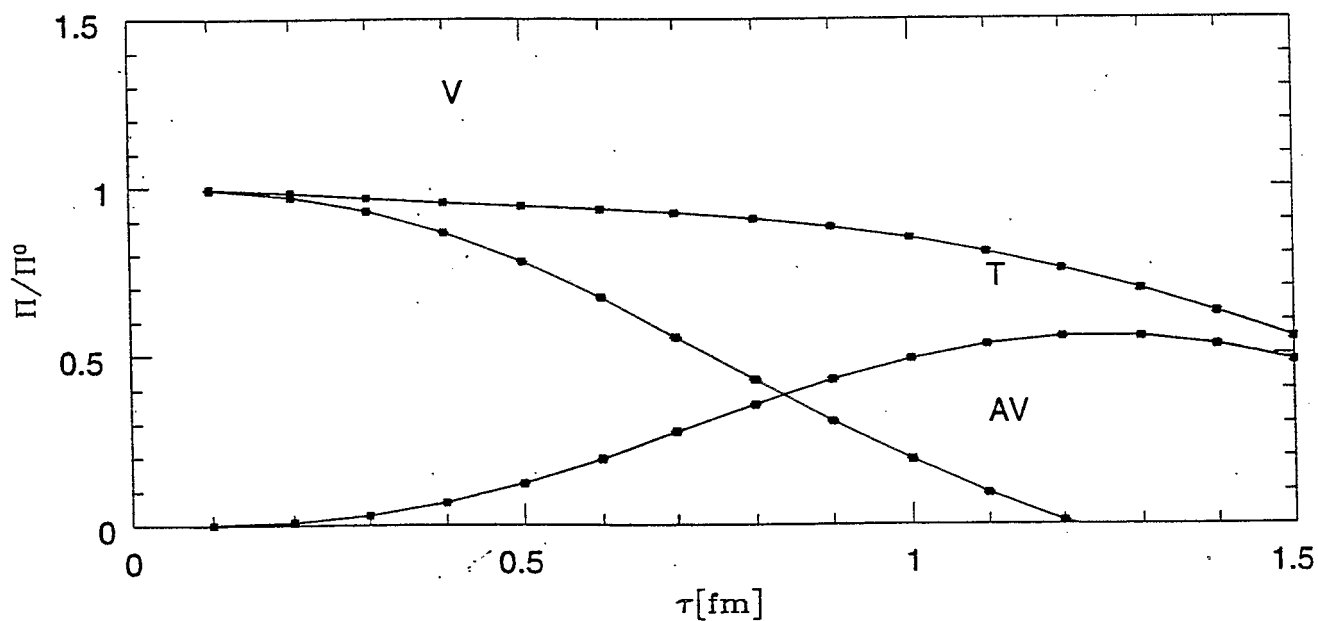
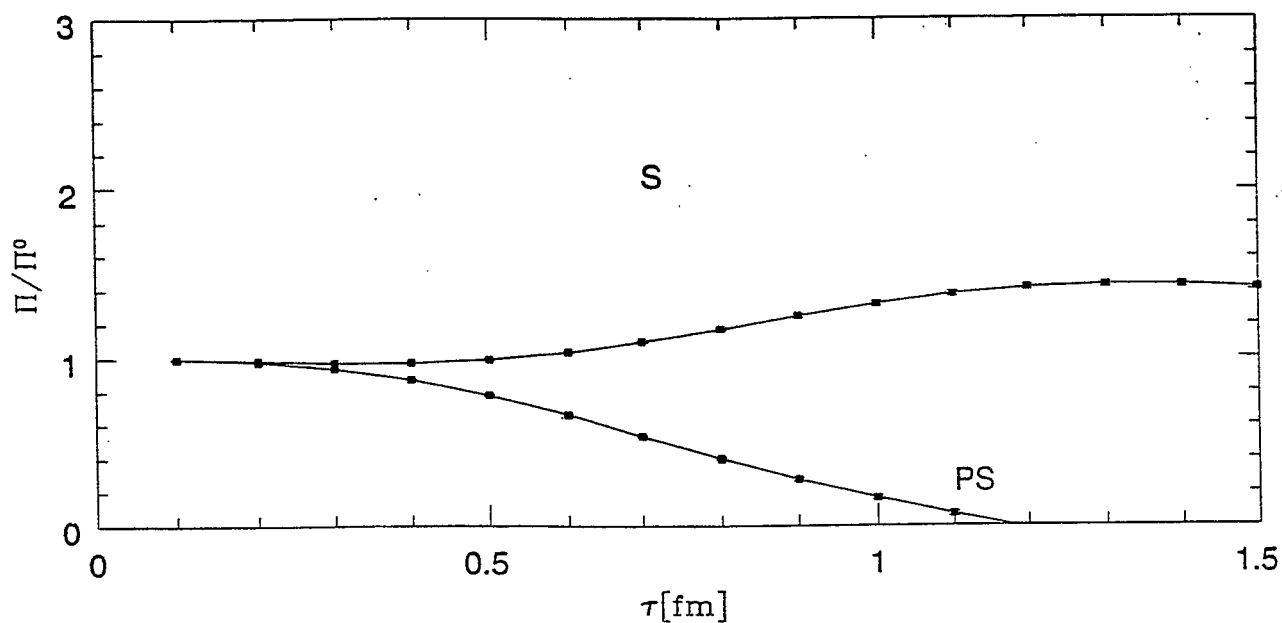
$$\Pi_r(x) = \langle j_r(x) U(x) j_r(0) \rangle$$

NON-STRANGE, COLOR 6 DIQUARKS



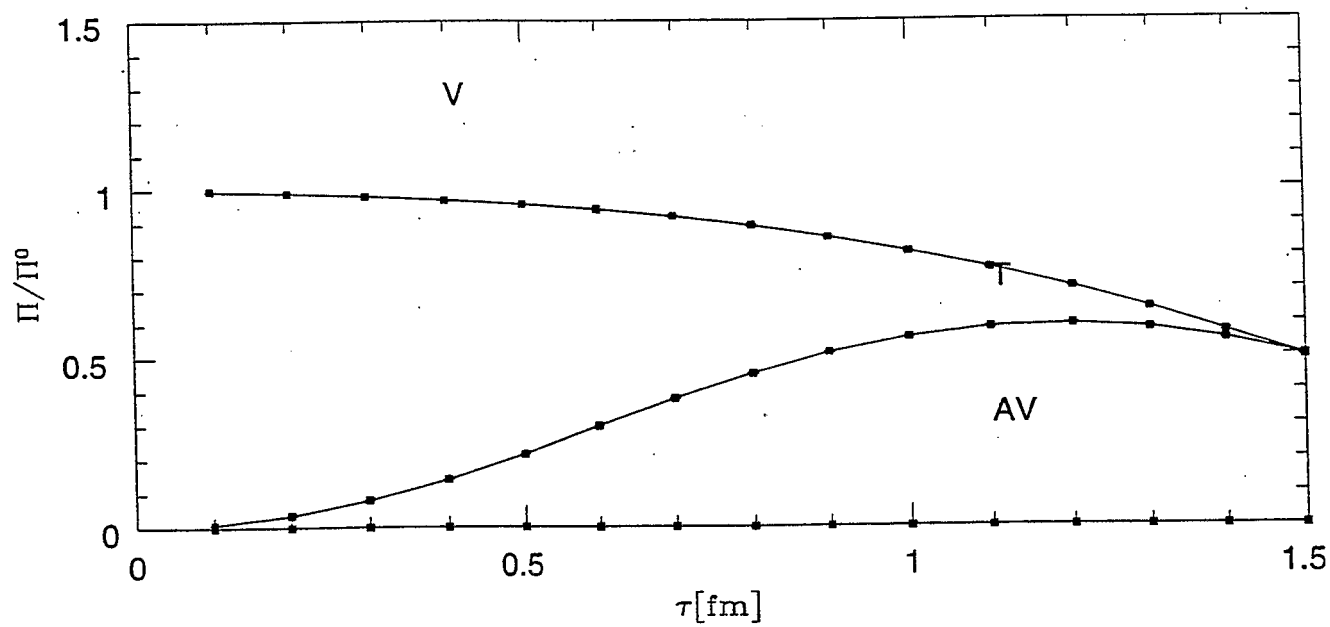
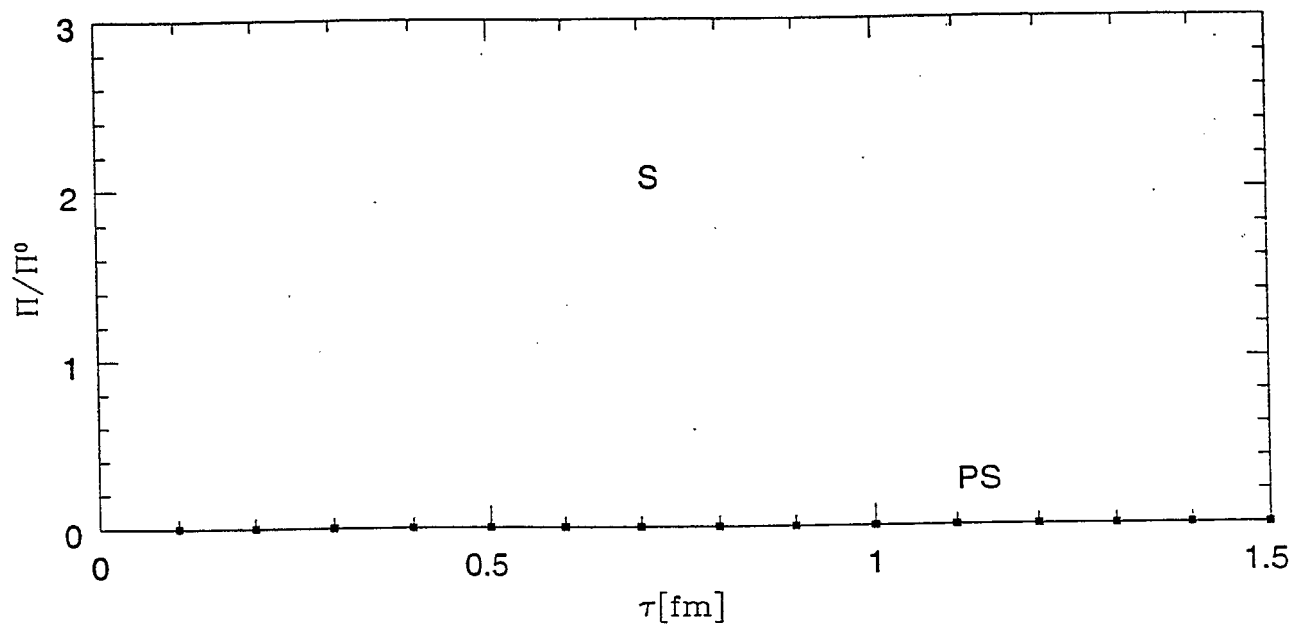
$$j_F^a(x) = \bar{q} C \Gamma \tau \lambda_s^a q$$

S=1, COLOR $\bar{3}$ DIQUARKS



$$J_\Gamma^a = u^b C \Gamma s^c \epsilon^{abc}$$

S=2, COLOR $\bar{3}$ DIQUARKS



$$J_T = \epsilon^{abc} s^b c^c s^a$$

CHIRAL SYMMETRY BREAKING, GAP EQU.

- PARTITION FUNCTION

$$Z = \frac{1}{\pi} \int [d\psi_i] u(\psi_i) \text{DET}(D + m_f)^{N_f}$$

- MEAN FIELD APPROXIMATION \Rightarrow GAP EQUATION

$$M(0) = \beta \int d^4k \frac{M(k)}{k^2 + M^2(k)} F(k) \text{ --- } = \text{ --- } + \text{ --- } \bigcirc$$

COUPLING β DETERMINED SELF-CONSISTENTLY

- QUARK CONDENSATE

$$\langle \bar{\psi}\psi \rangle = 4N_c \int \frac{d^4k}{(2\pi)^4} \frac{M(k)}{k^2 + M^2(k)} \approx -(230 \text{ MeV})^3$$

- RPA MODES

$$\Pi_T(q) = \Gamma(q) \frac{1}{1 + G(q)} \Gamma(q)$$

- MICROSCOPICALLY: LOW-LYING MODES BECOME COLLECTIVE

$$\langle \bar{\psi}\psi \rangle = -\frac{1}{\pi} u(\lambda=0)$$

GAP EQUATION WITH QQ CONDENSATES

- NAMBU-GORLON PROPAGATOR, $\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^T \end{pmatrix}$

$$G^{-1} = \begin{pmatrix} S^{-1} & \Delta \\ \bar{\Delta} & (S^{-1})^T \end{pmatrix}$$

- SCHEM. MODEL: SELF ENERGIES

$$S^{-1} = \not{p} - M \quad \Delta = \Delta_0 \cos \tau_2 \lambda_2$$

- EFF. POTENTIAL (MFA)

$$\Gamma = -\frac{1}{2} \text{tr} \log(G) + \text{tr}(G\Sigma) + g_M \text{tr}(S)^2 + g_D \text{tr}(\Delta) \text{tr}(\bar{\Delta})$$

- GAP EQUATIONS

$$M = g_M \int d^4p \frac{M}{p^2 - M^2} \left(1 + \frac{\Delta^2}{p^2 - (M^2 + \Delta^2)} \right)$$

$$\Delta = g_D \int d^4p \frac{\Delta}{p^2 - (M^2 + \Delta^2)}$$

- FIND ($\mu=0$): $\Delta \neq 0$ POSSIBLE, BUT NOT THE MINIMUM OF V FOR $N_c > 2$

$$\text{Feynman diagram: } \text{fermion line} = \text{fermion line} + \text{fermion line with self-energy insertion} + \text{fermion line with self-energy insertion and loop diagram}$$

$$\text{Feynman diagram: } \text{scalar line} = \text{scalar line} + \text{scalar line with self-energy insertion} + \text{scalar line with self-energy insertion and loop diagram}$$

FINITE DENSITY GAP EQU.

- $\overline{Q}Q$ GAP EQUATION ($\Delta=0$)

$$\mu = \mu_0 + 4G_1 |\langle \overline{\psi}\psi \rangle|$$

$$\Rightarrow \mu_{\text{crit}} = (2-\epsilon) \mu_0$$

$$\langle \overline{\psi}\psi \rangle = -\frac{3}{\pi^2} \int_0^\mu p^2 dp \frac{\mu}{E_p} (1 - u_q(p) - u_{\overline{q}}(p))$$

- $Q\overline{Q}$ GAP EQUATION ($\mu \ll \Delta \ll \mu$)

$$\Delta = G \frac{8}{(2\pi)^2} \int_{\mu-\lambda}^{\mu+\lambda} p^2 dp \frac{\Delta}{((E_p - \mu)^2 + \Delta^2)^{1/2}}$$

LOG SINGULARITY

\Rightarrow NO CRIT. COUPLING

GAP DETERMINED BY

$$\Delta \sim \lambda$$

FORM FACTOR

$$\Delta \sim \exp\left(-\frac{1}{\left(\frac{d\mu}{d\epsilon}\right)G_1}\right)$$

DENSITY OF STATES, COUPLING

$$G \sim \exp(-N_f p^2 (\mu - \mu_c)^2)$$

SCREENING

$$\Rightarrow \Delta \sim (50-100) \text{ MeV}$$

- FREE ENERGY

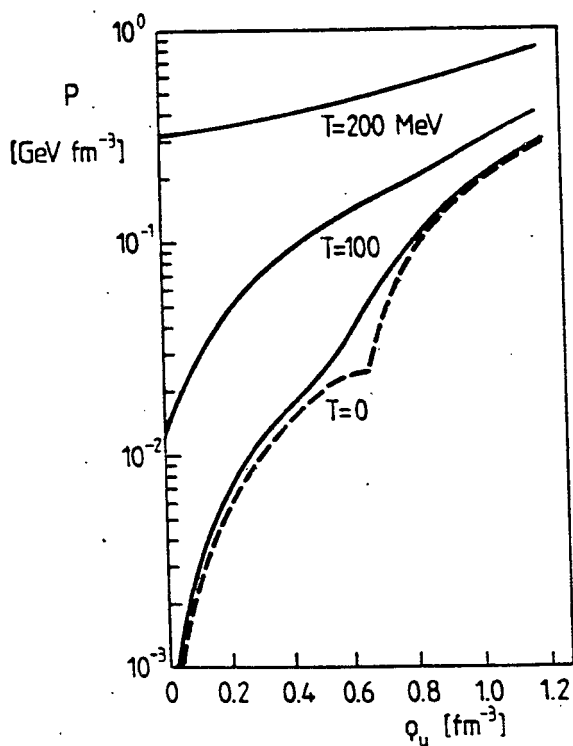
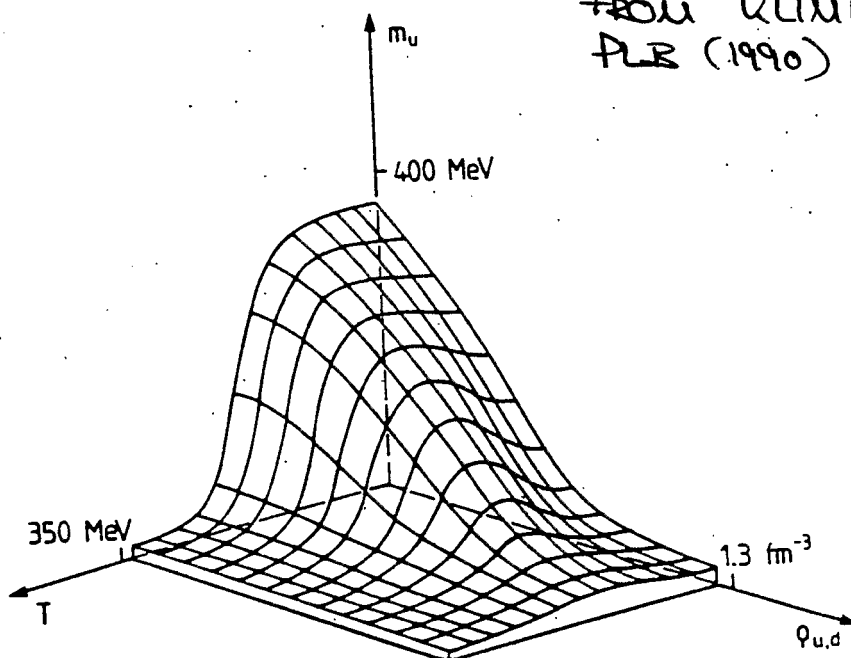
$$F \sim -\frac{1}{2} \frac{\mu p_F}{\pi^2} \Delta^2 + \dots$$

- CRITICAL TEMPERATURE

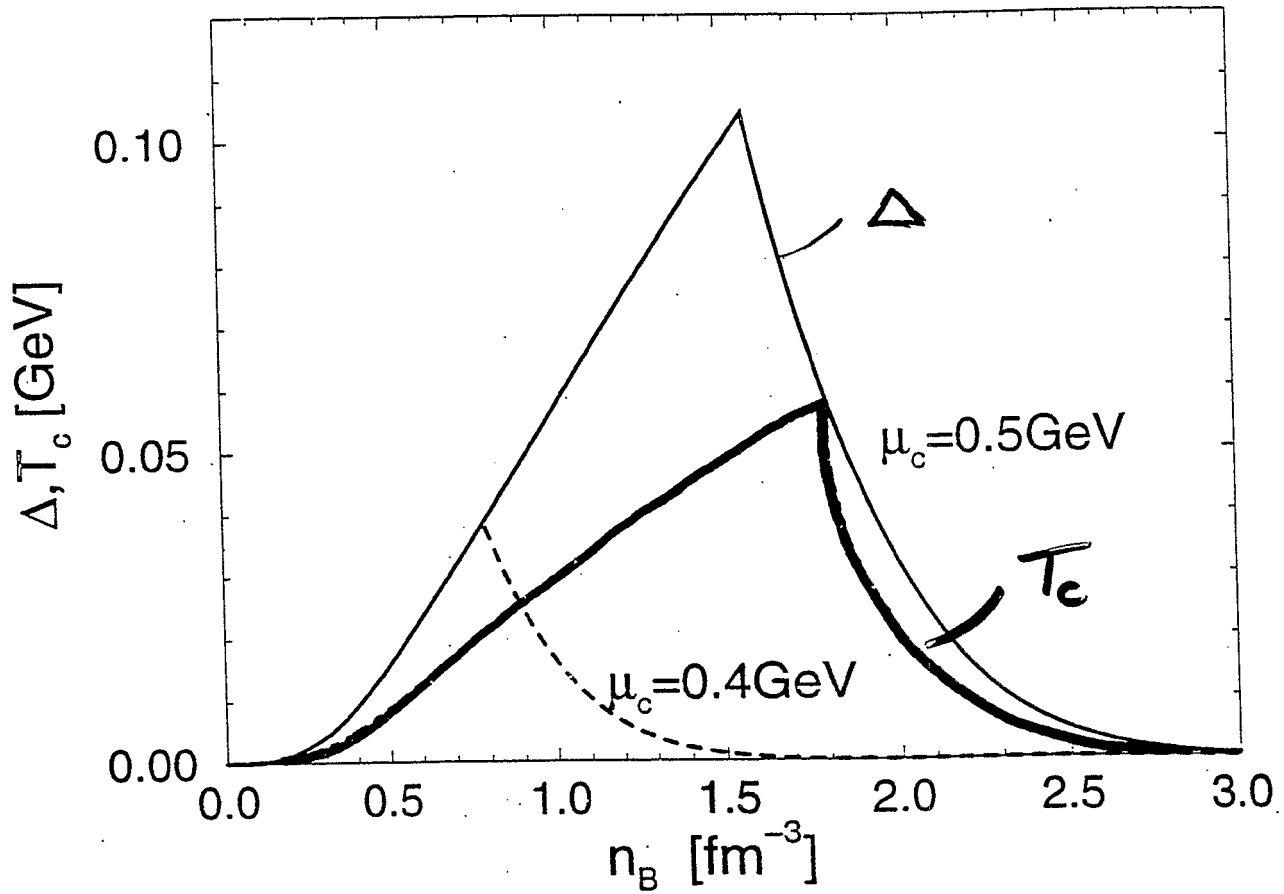
$$T_c \simeq \frac{\epsilon}{2} \Delta(T=0) \simeq 0.57 \Delta(T=0)$$

NJL THERMODYNAMICS

FROM KLINT ET AL
PLB (1990)

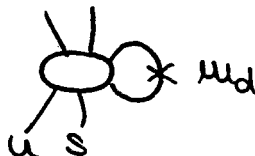
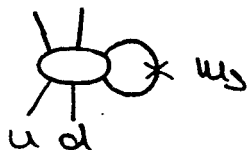


BCS GAP EQUATION FOR SCALAR DIQUARKS



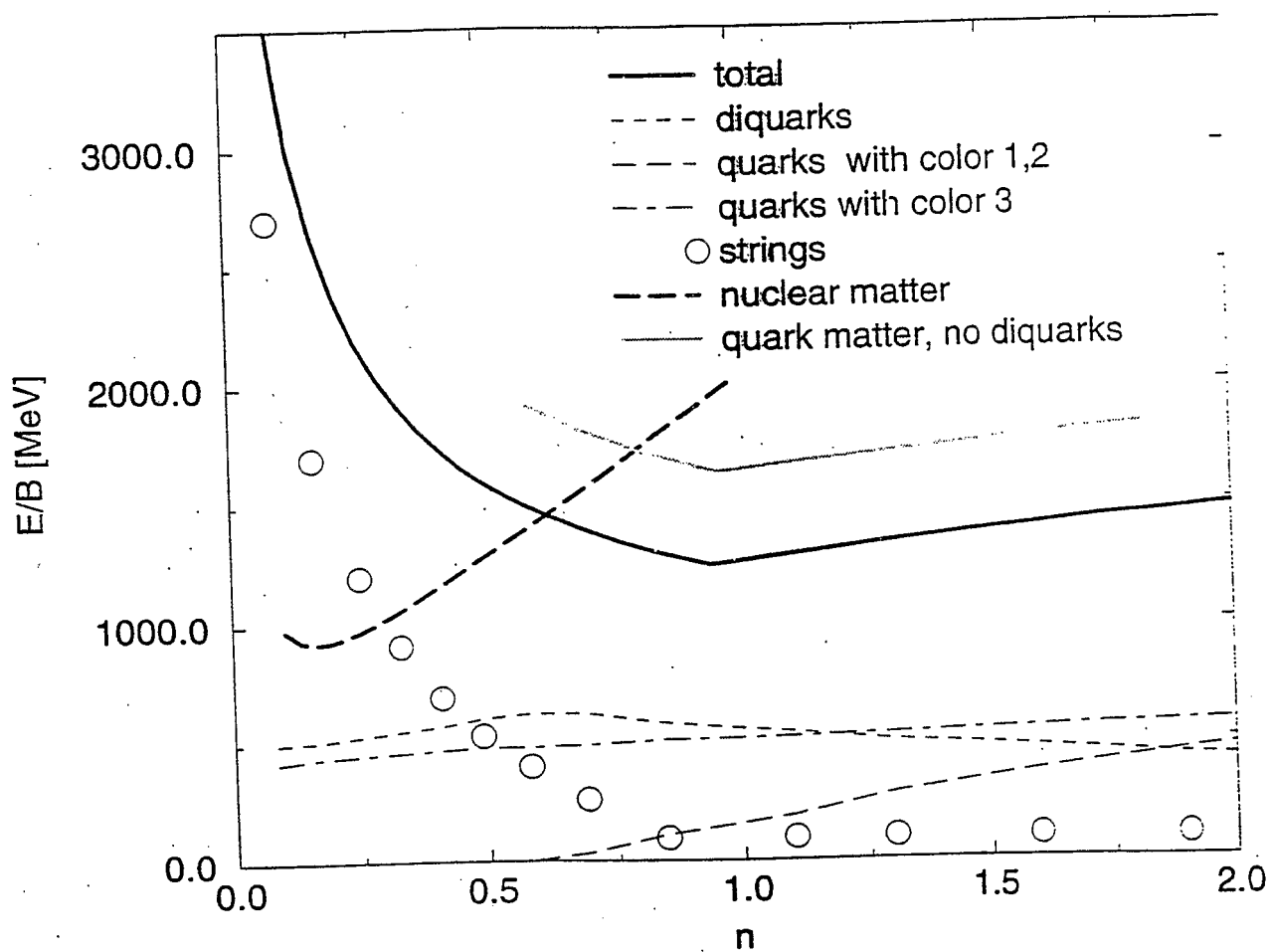
BEYOND $(u\gamma_s\lambda^A d)$ CONDENSATION

- SYMMETRIC QUARK MATTER $\Rightarrow u_3 d_3$ LEFTOVER (+S QUARKS)
- $(u\gamma_s\lambda^A s)$ PAIRING? COUPLING SMALL



- $(u\gamma_{\mu\nu}\lambda^S d), (u\gamma_{\mu\nu}\lambda^S s)$: COLOR 6 PAIRING IN VECTOR CHANNEL? (ARW)
- $(s\gamma_{\mu\nu}\lambda^A s)$: PERT. PAIRING IN VECTOR CHANNEL? (BL)
- FROM NEUTRON SUPERFLUIDITY TO COLOR SINGLET 6Q CONDENSATION? $(udd)^2, (uds)^2, \dots$?
- DECONFINEMENT: CONFINED \rightarrow HIGGSED, (CONF/HIGGS), ...
- BCS OR NOT? IDEAL BCS: GAP EXP. SMALL, QUASI-PARTICLES LARGE, NO PRE-FORMED PAIRS
HERE: GAP LARGE, PAIRS COMPACT

SCHEMATIC EQUATION OF STATE FOR QUARK-DIQUARK MATTER



INSTANTONS AT FINITE μ

- PARTITION FUNCTION

$$Z = \sum_{N_+, N_-} \frac{1}{N_+! N_-!} \int \prod_i [d\psi_i] e^{-S} \prod_f \frac{N_f}{f} (\text{DET}(i\mathcal{D} + i\mu\gamma_4 + i m_f))$$

COMPLEX
↙ ↘
SIGN PROBLEM

- $U_A(1)$ ANOMALY \Rightarrow ZERO MODES (CARVALHO, ABRAMOSON, ...)

$$(i\mathcal{D} + i\mu\gamma_4)\psi_0(\mu, x) = 0$$

- LARGE $\mu \Rightarrow$ SCREENING

$$u(p) \sim u(p, \mu=0) \exp(-N_f \mu^2 p^2) \quad (\mu > \mu_c)$$

- DYNAMICS: $(i\mathcal{D})$ in ZERO MODE BASIS

$$(i\mathcal{D} + i\mu\gamma_4)_{12} = \begin{pmatrix} 0 & T(\mu) \\ T^\dagger(-\mu) & 0 \end{pmatrix} \quad \text{NOT HERM.}$$

OVERLAP MATRIX ELEMENTS

$$T_{1A}(\mu, z) = \int dx \psi^\dagger(x-z) (i\mathcal{D} + i\mu\gamma_4) \psi(x)$$

ANISOTROPIC

$$T_{1A}(\mu, z_R) \sim \mu \sin(\mu z) / z^2 \quad \text{OSCILLATING} \quad \left(\begin{array}{c} \text{SHARP FERMION} \\ \text{SURFACE } \psi_0 \end{array} \right)$$

$$T_{1A}(\mu, z_L) \sim \mu^2 / z^4 \quad \text{LONG RANGE} \quad \rightarrow \cdot \exp(-\Delta z) \cdot \text{GAP } \psi_0$$

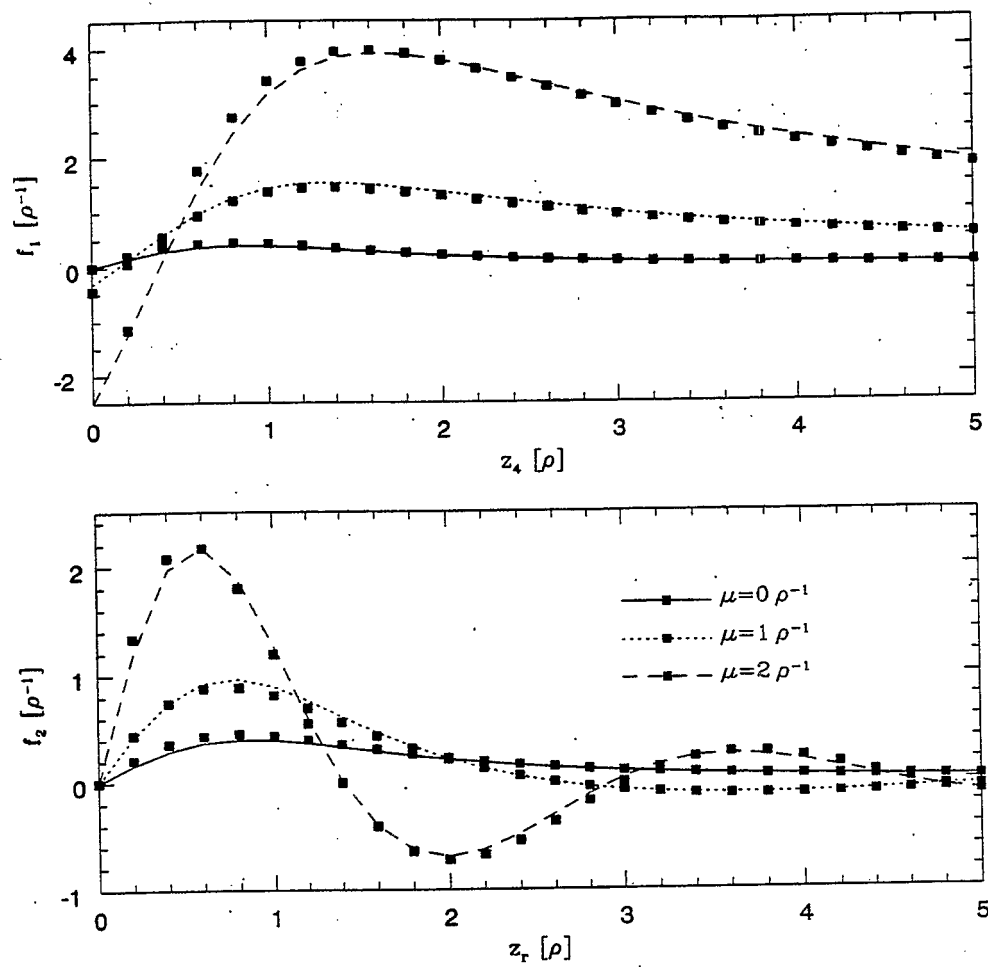


FIG. 1.

SIMPLE THEORY: $N_c = N_f = 2$

- THEORY HAS $Q \leftrightarrow \bar{Q}$ SYMMETRY (PG-SYM)

$$\langle \bar{\psi} \psi \rangle \leftrightarrow \langle \epsilon^{ab} \psi^a \psi^b \rangle \quad \text{PIONS} \leftrightarrow \text{SC. DIQU.} (\hat{=} \text{BARYONS})$$

$$V_{\text{eff}} = \lambda (\sigma^2 + \pi^2 + S^2 + \bar{S}^2 - v^2)^2 - A\sigma + \mu^2(S^2 + \bar{S}^2)$$

$$A \neq 0, \mu = 0 \quad \langle \sigma \rangle \neq 0 \quad \mu > \mu_{\text{crit}} \quad \langle S \rangle = \langle \bar{S} \rangle \neq 0 \quad \langle \sigma \rangle \rightarrow 0$$

- EASY TO SIMULATE:

$$(i\not{D} + i\mu\gamma_4) \psi_i = \lambda_i \psi_i \quad (\pm \lambda_i, \pm \lambda_i^*)$$

$$\Rightarrow \text{DET}(i\not{D} + i\mu\gamma_4) = \prod_i \lambda_i \quad \underline{\text{REAL}}$$

$$N_f = 2: (\text{DET}(i\not{D} + i\mu\gamma_4))^2 > 0$$

- NOTE: $N_c = 3$ WITH $|\text{DET}(\dots)|$ BEHAVES SIMILAR

$$Z = \int \mathcal{D}A \, e^{-S} \underbrace{\text{DET}(i\not{D} + i\mu\gamma_4)}_{q(\mu), \bar{q}(-\mu)} \underbrace{\text{DET}(i\not{D} - i\mu\gamma_4)}_{\bar{Q}(-\mu), Q(+\mu)}$$

$$\Rightarrow \text{LIGHTEST } B \neq 0 \text{ STATE } \pi = \bar{Q}(\mu) q(\mu)$$

$$\Rightarrow \mu_{\text{crit}} \gtrsim m_{\pi/2} \sim m_q^{1/2}$$

$\mu \neq 0$ SIMULATIONS, $N_c = 2$

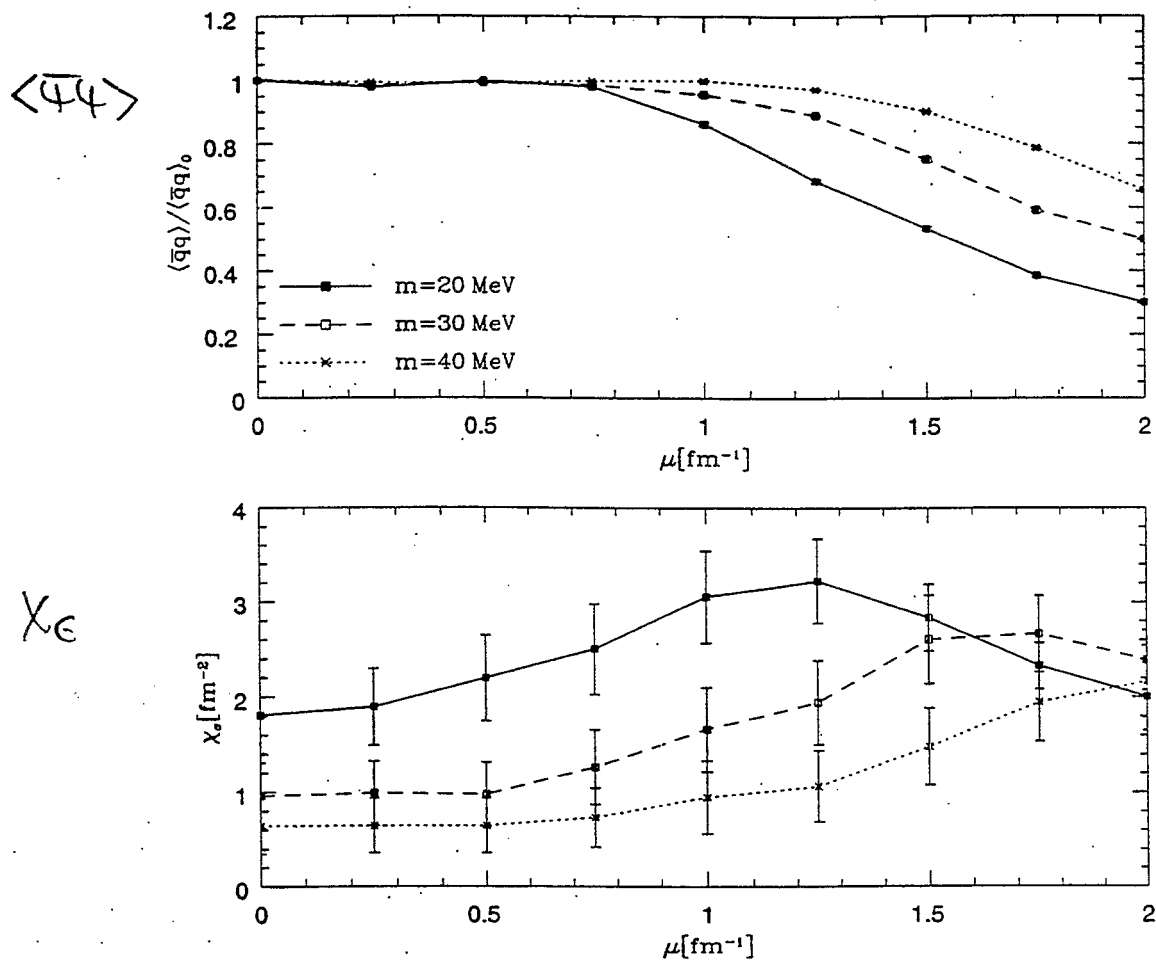
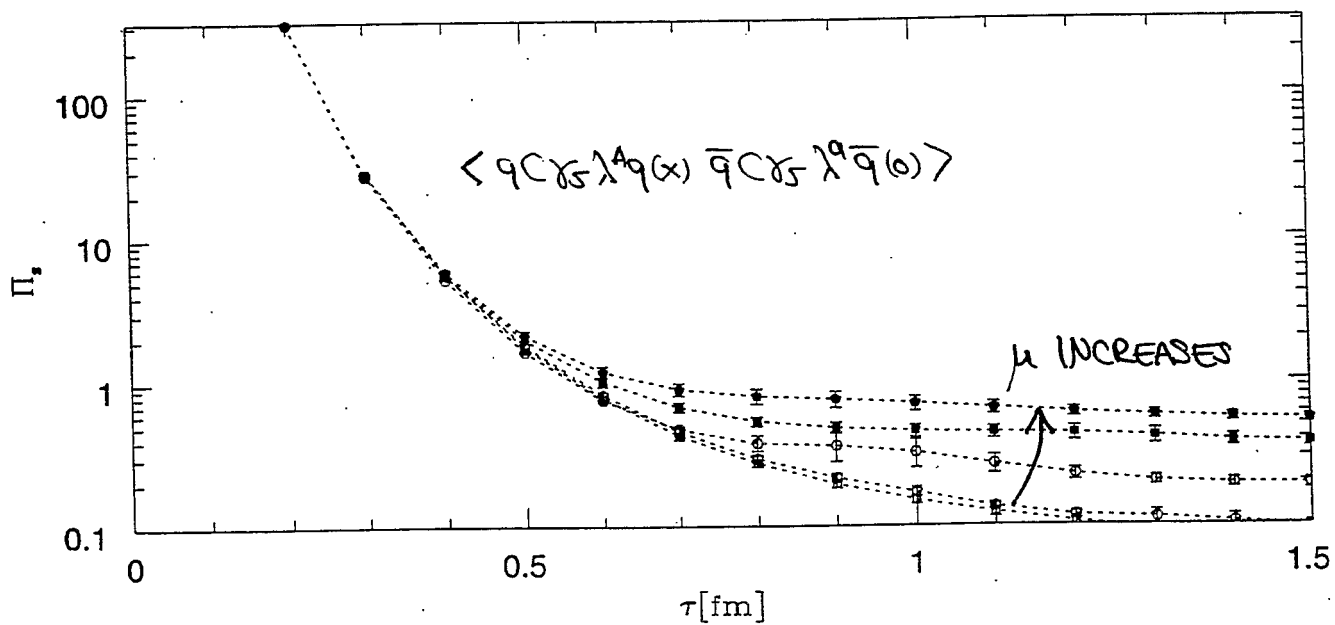
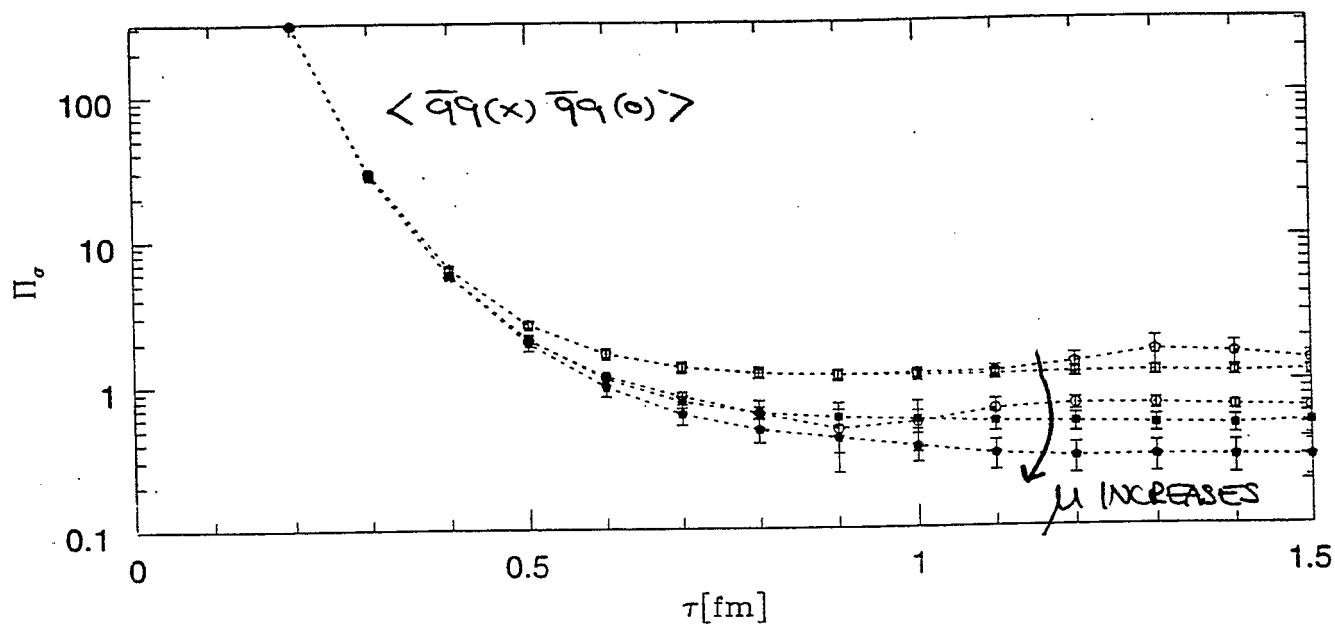


FIG. 5.

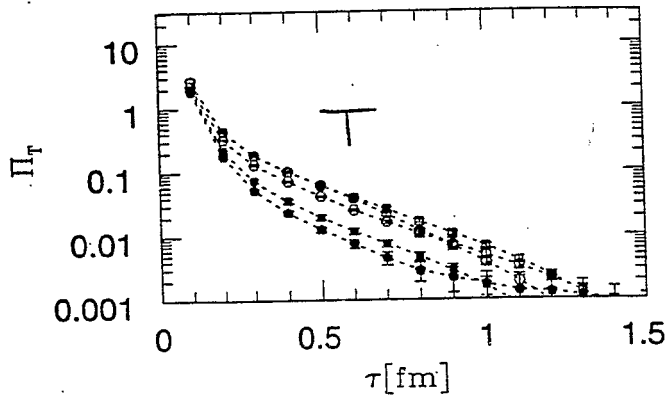
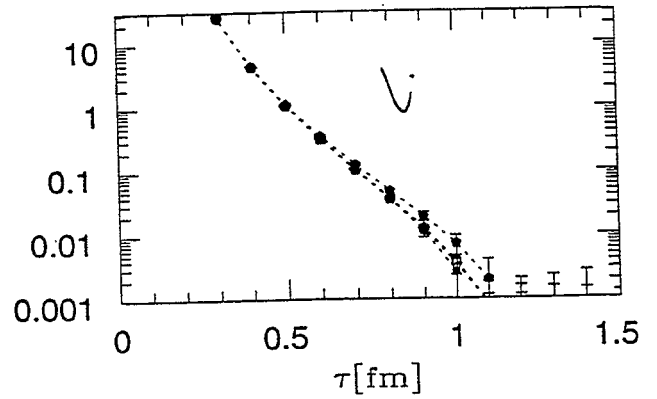
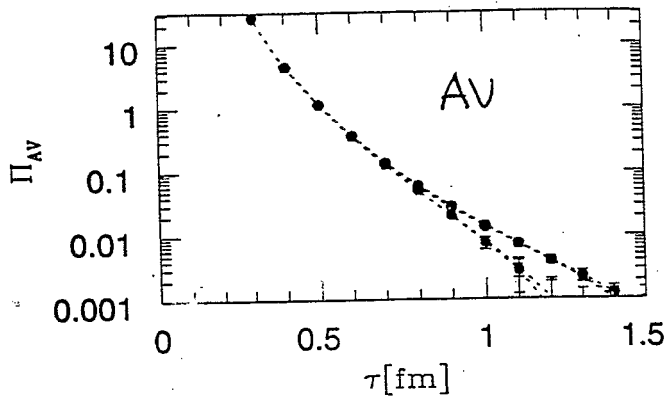
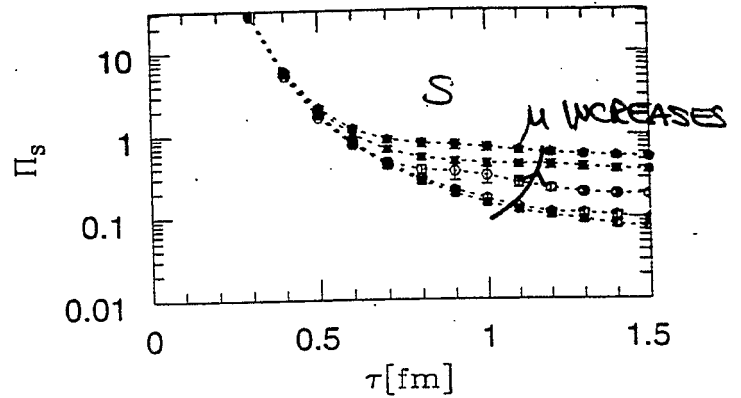
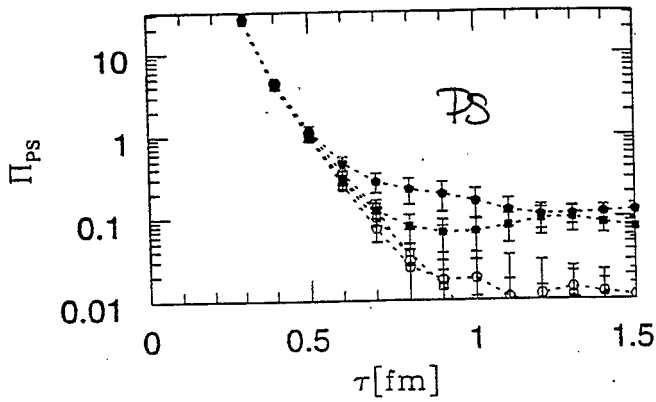
→ CRITICAL $\mu \rightarrow 0$ AS $m \rightarrow 0$

$\bar{Q}Q$ VERSUS QQ CONDENSATION

$N_c=2$ SIMULATIONS



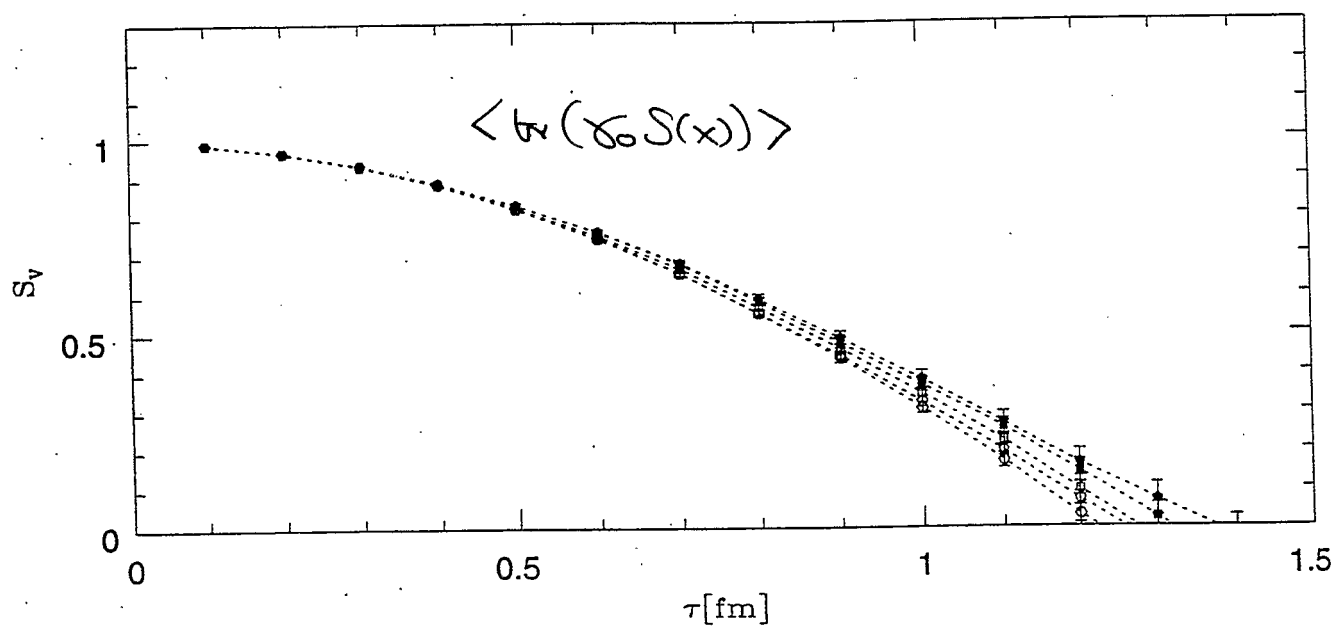
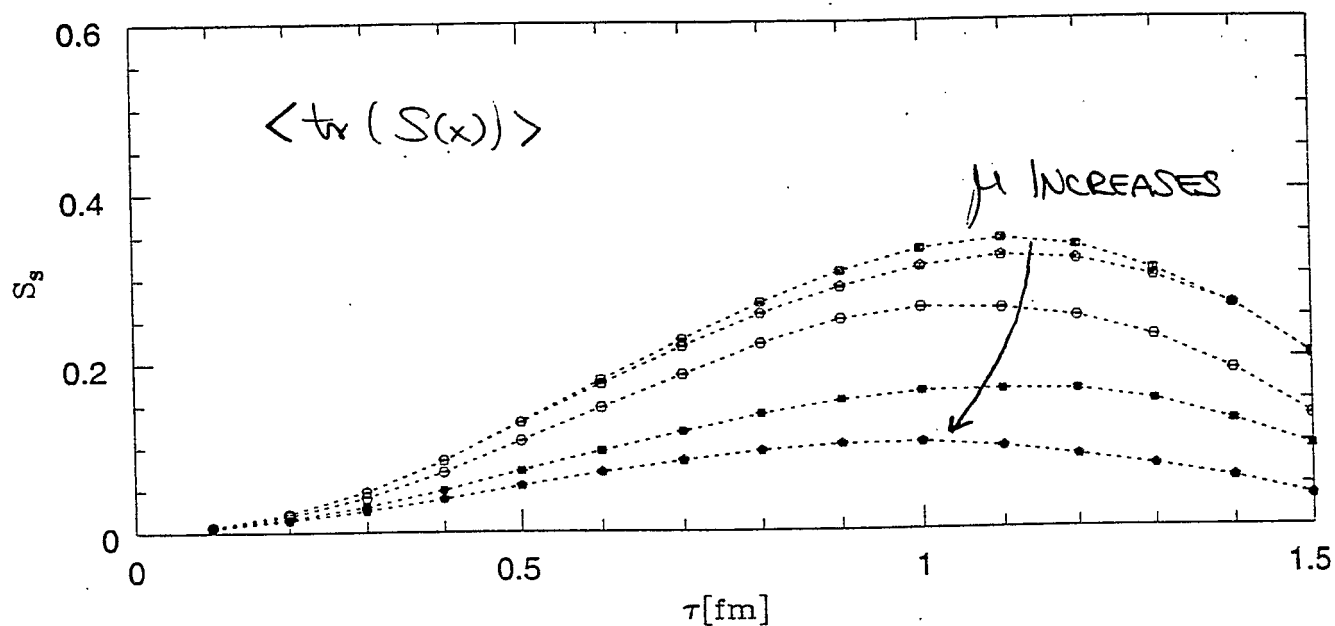
DIQUARK CORRELATORS AT $\mu \neq 0$



$N_c = 2$ SIMULATIONS

QUARK PROPAGATOR

$N_c=2$ SIMULATIONS



HARD THEORY: QCD ($N_c=3$)

- EXPECT:

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}\psi \rangle_0 \quad \mu < \mu_c \approx \frac{\mu_B}{3} \quad (\approx \frac{\mu_D}{2})$$

- SIGN PROBLEM \leadsto APPROXIMATIONS (?)

A) QUENCHED

$$Z = \int dA e^{-S} \quad \text{[Diagram: A circle with a vertical axis and a horizontal axis, both labeled with a lambda symbol. The circle is filled with diagonal lines.]}$$

$$\mu_c \approx \frac{\mu\pi}{2} \quad \downarrow$$

B) MODULUS OF DETERMINANT

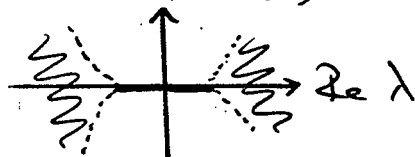
$$Z = \int dA e^{-S} |\text{DET}(i\mathcal{D} + i\mu\gamma_5)| \quad \mu_c \approx \frac{\mu\pi}{2} \quad \downarrow$$

3) INCLUDE PHASE IN AVERAGES

$$\langle \bar{\psi}\psi \rangle_{||} \equiv \int dA (\bar{\psi}\psi) e^{-S} |\text{DET}(i\mathcal{D} + i\mu\gamma_5)|$$

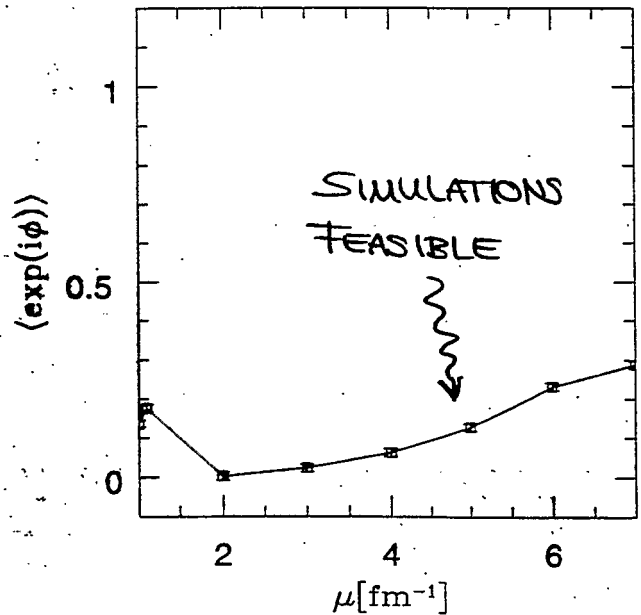
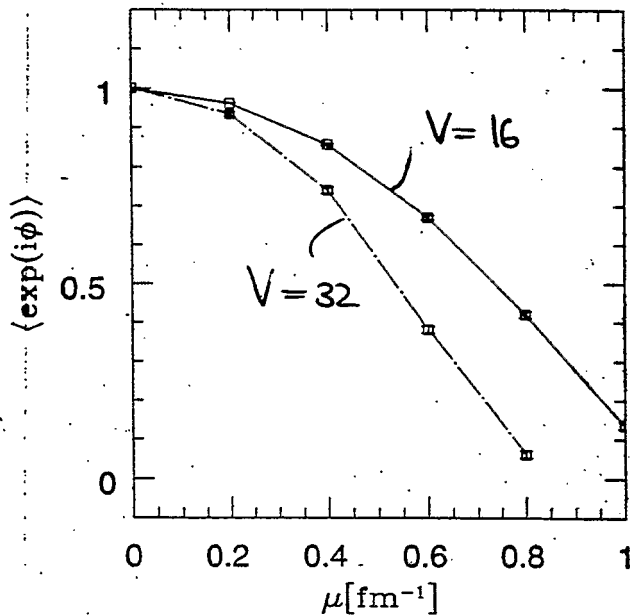
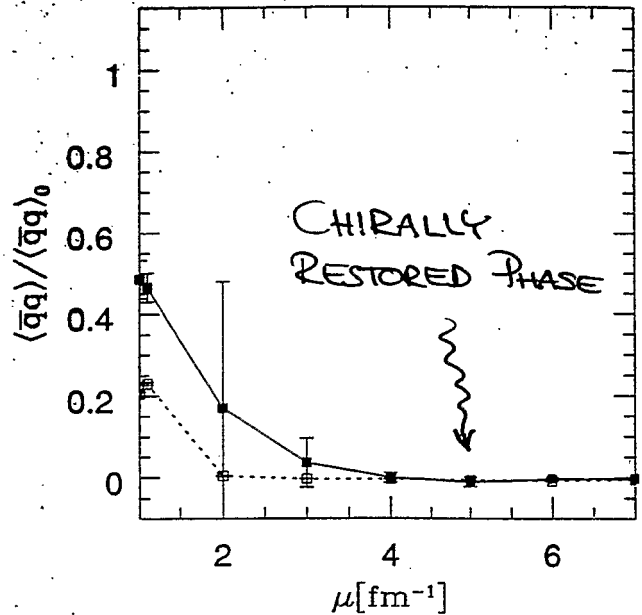
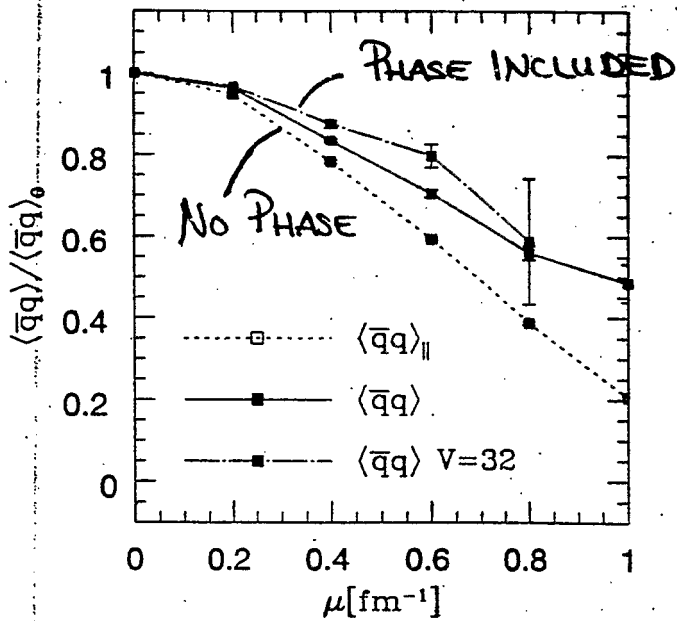
$$\leadsto \langle \bar{\psi}\psi \rangle = \frac{\langle \bar{\psi}\psi e^{i\phi} \rangle_{||}}{\langle e^{i\phi} \rangle_{||}} \quad \leftarrow \text{TROUBLE } \nabla$$

EXPECT: $\langle \bar{\psi}\psi \rangle = \text{CONST} (\mu < \mu_c)$, SMALL λ REAL



D) OTHER METHODS? CANONICAL, ...

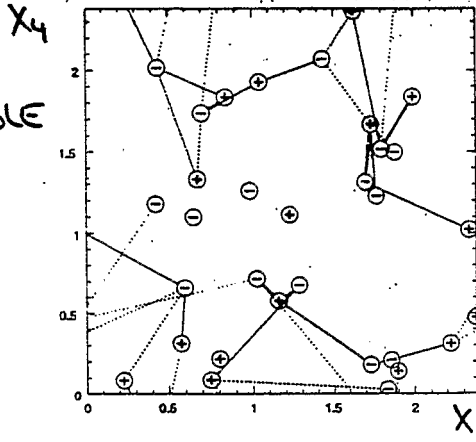
$\mu \neq 0$ SIMULATIONS



- UNQUENCHED
- BASIS OF EXACT ($\mu \neq 0$) ZERO MODES

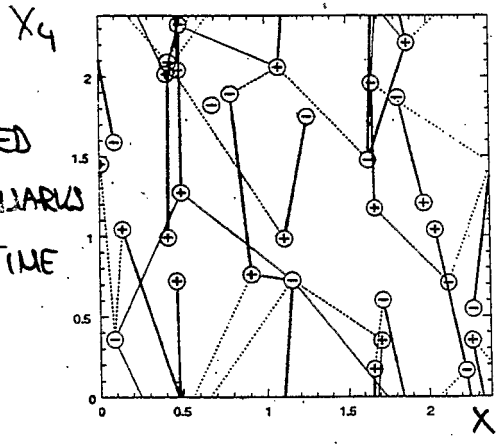
TYPICAL INSTANTON CONFIGURATIONS

RANDOM ENSEMBLE
 $\langle \bar{\psi}\psi \rangle \neq 0$



$$\mu = 0$$

BONDS SATURATED
 BY VALENCE QUARKS
 PROPAGATING IN TIME
 DIRECTION
 $\Rightarrow \langle \bar{\psi}\psi \rangle = 0$

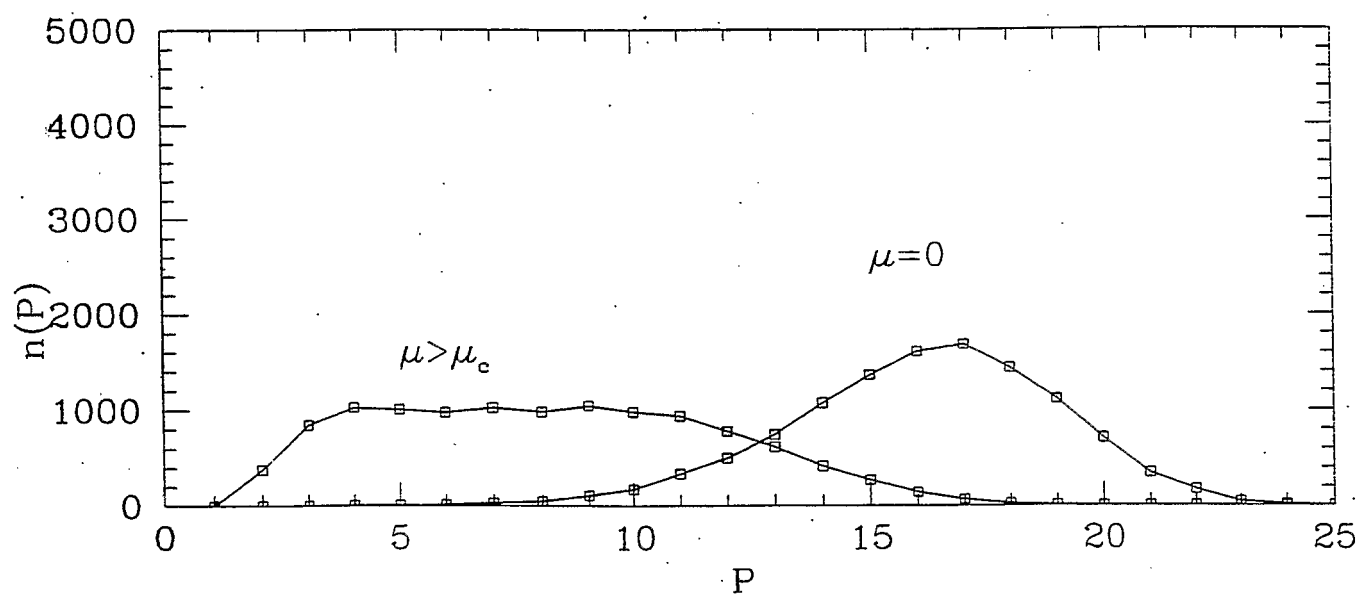
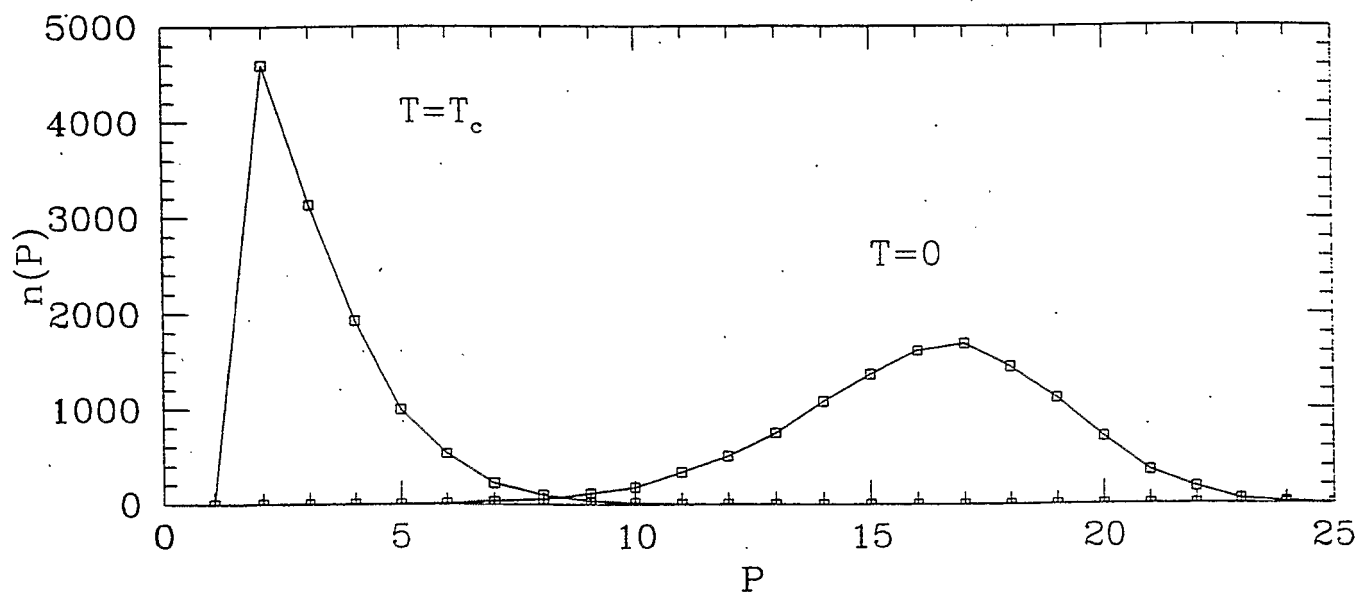


$$\mu = 3\Lambda$$

FIG. 9.

==== STRENGTH OF FERMIONIC
 "HOPPING" MATRIX ELEMENTS
 $T_{1A} = \int dx \bar{\psi}_\pm (iD) \psi_A$

COLLECTIVITY OF EIGENSTATES



PARTICIPATION NUMBER

$$P = 1 / \sum |c_i|^4 \quad \psi = \sum c_i \psi_i$$

Test of Discrete Symmetries in Relativistic Heavy Ion Collisions

Miklos Gyulassy¹, T.D. Lee¹ and Yang Pang^{1,2}

¹*Department of Physics, Columbia University, New York, NY 10027*

and

²*Department of Physics, Brookhaven National Laboratory, Upton, NY 11973*

For matters at high baryon densities, such as those created in relativistic heavy ion collisions at BNL-AGS, it has recently been shown by M. Alford, K. Rajagopal, and F. Wilczek [1], and also independently by R. Rapp, T. Schafer, E.V. Shuryak, and M. Velkovsky [2], that interesting condensates could be formed. The possibility for detecting the existence of some condensates from parity violation measurements was also suggested [3, 4].

Several explicit models [5] were constructed by T.D. Lee for studying the formation of condensates that spontaneously break the discrete symmetries. These models could provide a basic framework for testing discrete symmetries in relativistic heavy ion collisions.

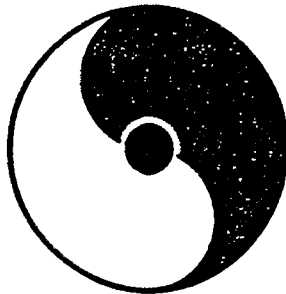
The longitudinal polarizations of Λ 's and $\bar{\Lambda}$'s are very effective in detecting parity violating condensates. The large number of Λ 's produced in relativistic heavy ion collisions at BNL-AGS, CERN-SPS and BNL-RHIC, suggests it is possible to make an accurate determination of the existence or absence of any parity violating condensates. Several experiments at BNL-AGS are capable of this type of measurement [6, 7].

The background for longitudinal polarization of Λ 's is dominated by Ξ decay to Λ . In fact, it was first suggested by M. Jacob to use the longitudinal polarization of Λ 's and $\bar{\Lambda}$'s as an indirect measurement of Ξ/Λ and $\bar{\Xi}/\bar{\Lambda}$ ratios under the assumption that parity is conserved in heavy ion collisions [8].

References:

- [1] M. Alford, K. Rajagopal, and F. Wilczek, hep-ph/9711395.
- [2] R. Rapp, T. Schafer, E.V. Shuryak, and M. Velkovsky, hep-ph/9711396.
- [3] F. Wilczek, in Proceedings of RIKEN BNL Research Center Symposium on *Non-Equilibrium Many Body Dynamics*, Volume 4 (1997).
- [4] F. Wilczek and K. Rajagopal, private communications.
- [5] T.D. Lee, Phys. Rep. 9C, (1974).
- [6] N. Samios, in this proceedings.
- [7] B. Cole, in this proceedings.
- [8] M. Jacob, Z. Phys. C38, 273 (1988).

**Test of Discrete Symmetries
in
Relativistic Heavy Ion Collisions**



Miklos Gyulassy, T.D. Lee and Yang Pang

**RIKEN BNL Research Center Workshop on
Color Superconductivity, Instantons, and Parity
(Non?)-Conservation at High Baryon Density**

November 11, 1997

Outline

- Spontaneous Breaking of Discrete Symmetries
- Observables from High Baryon Density
- Test of Discrete Symmetries

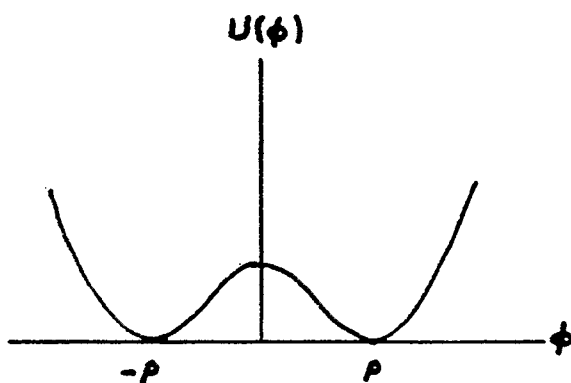
Spontaneous Breaking of T invariance

Model Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - U(\phi) - \bar{\psi}(\gamma^\mu\partial_\mu + m)\psi - ig\bar{\psi}\gamma_5\psi\phi$$

T.D. Lee, Physic Report 9C, (1974)

where $U(\phi) = \frac{1}{8}\kappa(\phi^2 - \rho^2)^2$



and $T\phi T^{-1} = -\phi$, $C\phi C^{-1} = \phi$, $P\phi P^{-1} = -\phi$

\mathcal{L} invariant under C , P and T

Vacuum expectation value $\langle \phi \rangle_{\text{vac}} = \rho > 0$

$$\phi = \rho + \delta\phi$$

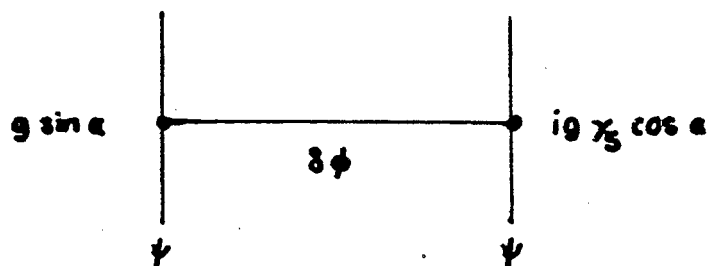
$$\psi \rightarrow e^{-i\frac{1}{2}\gamma_5\alpha}\psi \quad \tan\alpha \equiv g\rho/m$$

$$\bar{\psi}(m + ig\rho\gamma_5)\psi \rightarrow \bar{\psi}M\psi \quad M \equiv (m^2 + g^2\rho^2)^{\frac{1}{2}}$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\delta\phi)^2 - U - \bar{\psi}(\gamma^\mu\partial_\mu + M)\psi$$

$$-ig\bar{\psi}(\sin\alpha + i\gamma_5\cos\alpha)\psi\delta\phi$$

\mathcal{L} violates T , P , and CP



T violating amplitude

Spontaneous Breaking of P invariance

Model Lagrangian

$$\mathcal{L} = \dots - \bar{\psi}(\gamma^\mu \partial_\mu + m)\psi - ig\bar{\psi}\gamma^\mu\gamma_5\psi A_\mu$$

A_μ : Axial vector or $\partial_\mu\phi$ with

$$\langle A_0 \rangle = a_0 \neq 0 \quad \text{and} \quad \langle \vec{A} \rangle = 0$$

Energy eigenvalues for ψ

$$(E^2 - m^2 - \vec{p}^2 - 2a_0\vec{\sigma} \cdot \vec{p} - a_0^2)u = 0$$

\mathcal{L} Breaks P and C

Pseudo-Scalar in PDG

Name	Mass (MeV)	$I^G(J^{PC})$
π^0	140	$1^-(0^{-+})$
η	547	$0^+(0^{-+})$
η'	957	$0^+(0^{-+})$
...		

Axial-Vector in PDG

Name	Mass (MeV)	$I^G(J^{PC})$
$a_1(1260)$	1230	$1^-(1^{++})$
$f_1(1260)$	1285	$0^+(1^{++})$
...		

Condensates from high temperature and high
baryon densities

Many things to choose from:

Nucleon currents $\bar{\psi}\Gamma T\psi$

Quark and diquark currents

Meson currents

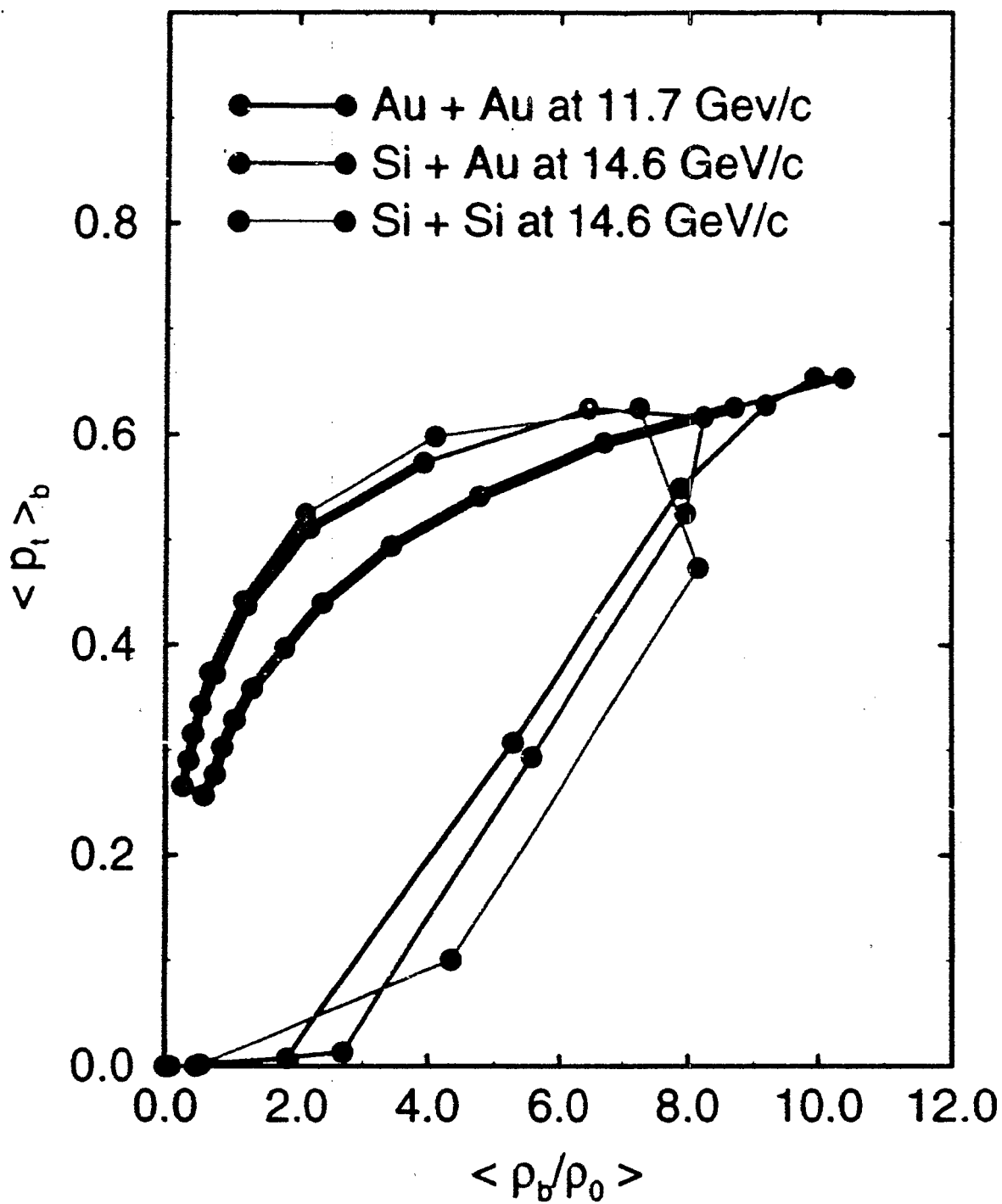
Gluons

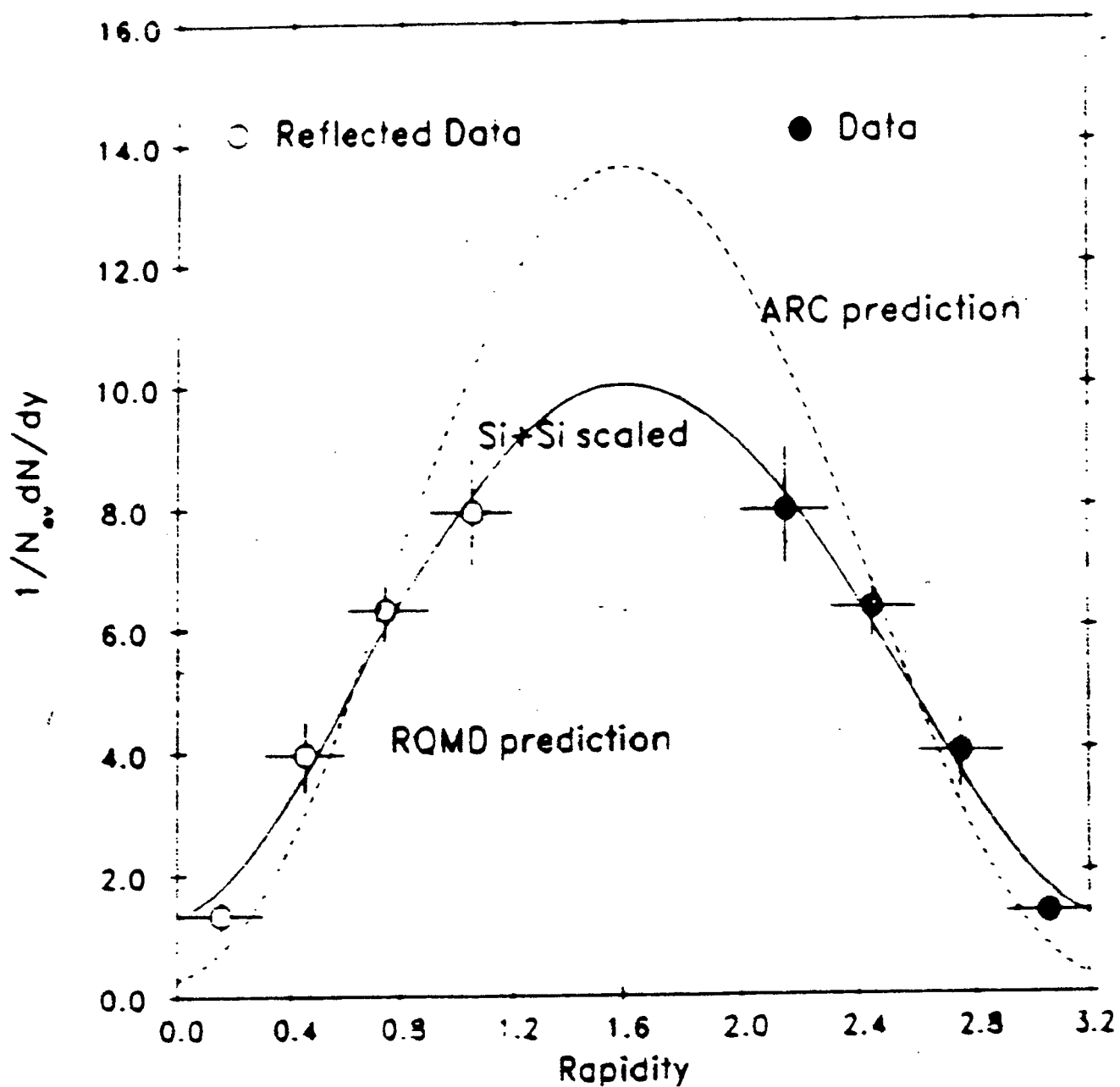
For example:

gluon + diquark (1^+) could form 0^{--}

Disoriented Chiral Condensate: 0^{-+}

T.J. Schlagel, et al.



Au on Au - Δ Rapidity

Decay of Λ with a definite helicity

$$M_{\Lambda \rightarrow \pi^- p} = a_s + a_p \vec{\sigma} \cdot \vec{k}$$

$$I(\theta) = \frac{1}{2}(1 + \alpha_\Lambda \cos \theta) \quad \alpha_\Lambda = \frac{2\text{Re}a_s^* a_p}{a_s^2 + a_p^2} = -0.64$$

Decay of Λ with polarization β

$$I(\theta) = \frac{1}{2}(1 + \alpha_\Lambda \beta \cos \theta)$$

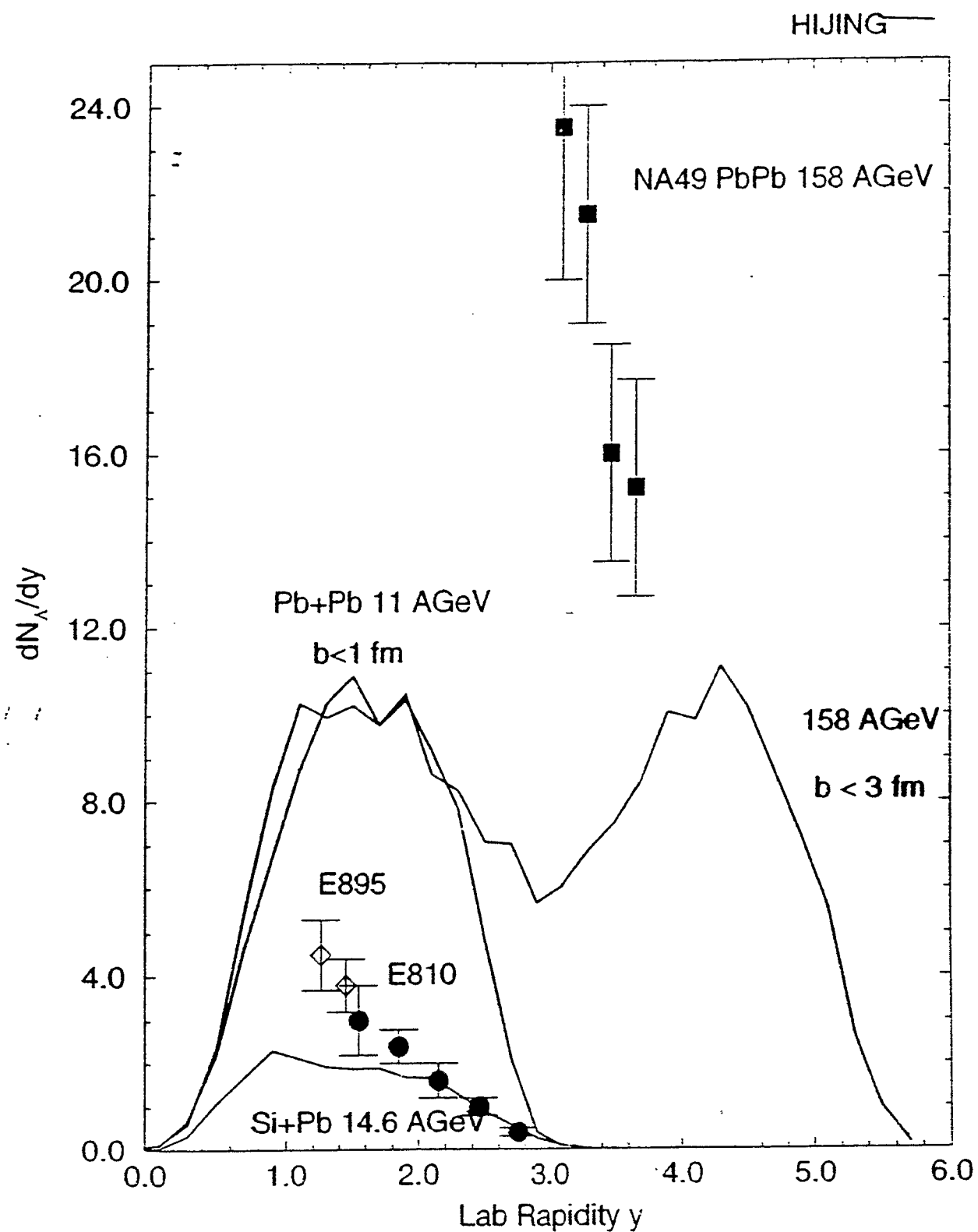
For spontaneous breaking of P in heavy ion collisions

A given event has fixed polarization: β

Average of β over events $= 0$

Average of β^2 over events $\neq 0$

Λ Production in A+B



$$I_{\Lambda\Lambda}(\theta_2 - \theta_1) = \frac{1}{2} \left(1 + \frac{2}{3} \alpha_{\Lambda}^2 \beta^2 \cos(\theta_2 - \theta_1) \right)$$

Single event measurement

	# of Λ	$\Delta\beta$	$\Delta\beta^2$
AGS Au + Au	15	0.3	0.1
SPS Pb + Pb	60	0.15	0.02
RHIC Au + Au	150?	0.1	0.01

Many events $10^6 \Lambda\Lambda$ $\Delta\beta^2 \sim 0.001$

Background from Ξ decay

$$\Xi \rightarrow \Lambda \pi \quad \alpha_{\Xi} = 0.44$$

$$\frac{\Xi}{\Lambda} \sim 0.1 \quad \beta_{\Xi} \sim 0.05$$

$$P_L(s) = \alpha_{\Xi} + \beta_{\Xi} \cos\theta$$

Conclusions:

- Test of discrete symmetries is a valuable tool for probing the existence of certain condensates, independent of any specific model
- Lambda polarization could be used to test parity violation from relativistic heavy ion collisions

Parity

N.P. SAMIOS

Brookhaven National Laboratory
Upton, New York 11973

Previous investigations of possible parity violations in strong and weak interactions is reviewed. The now classic studies of Λ^0 produced in π^-p interactions is discussed. Parity violation in Λ^0 decay (i.e. in weak interaction) was observed via an updown asymmetry, that is with respect to the production plane defined by $\hat{p}_{\pi^-} \times \hat{p}_{\Lambda^0}$. If parity is violated in the strong interaction, that would give rise to asymmetries in the production plane, categorized by a front-back or left-right asymmetry. No such effects were found in Λ^0 production. Similar studies can be carried out for Λ^0 production in Gold-Gold reactions. In this case one can look for a parity violating effect in strong interactions via asymmetries in the production plane defined by $\hat{p}_{Au} \times \hat{p}_{\Lambda}$. It would also be interesting to look for parity violation in the weak interaction in Λ^0 produced by AuAu interactions. Such possibilities for various AGS experiments were also discussed.

Parity.

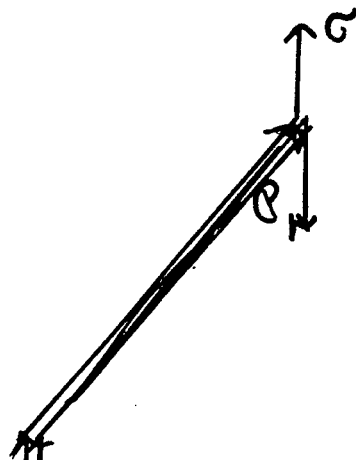
Strong Interactions

Weak Interactions

Λ^0 's.

M.P. Samios

Nov. 11, 1997



Parity Cons. in Strong Int.

π^- No Polarization in production plane.

$\pi^- p \rightarrow \Lambda^0 \theta^0$ Polarization \perp to Prod. Plane.

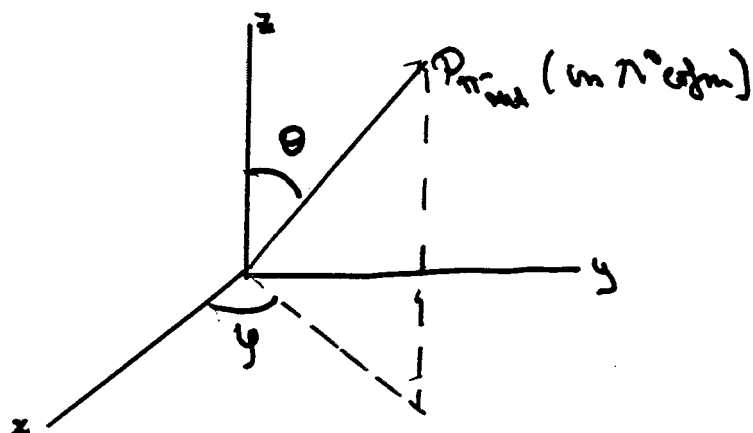
Standard Coord. System:

\hat{z}_{axis} normal to Production plane.

$$\vec{P}_{\pi^+} \times \vec{P}_{\Lambda^0}$$

$$\pi^- p \rightarrow \Lambda^0 + \theta^+$$

$$\hookrightarrow p + \pi^-_{\text{out}}$$



θ . up down angle. Parity - weak int.

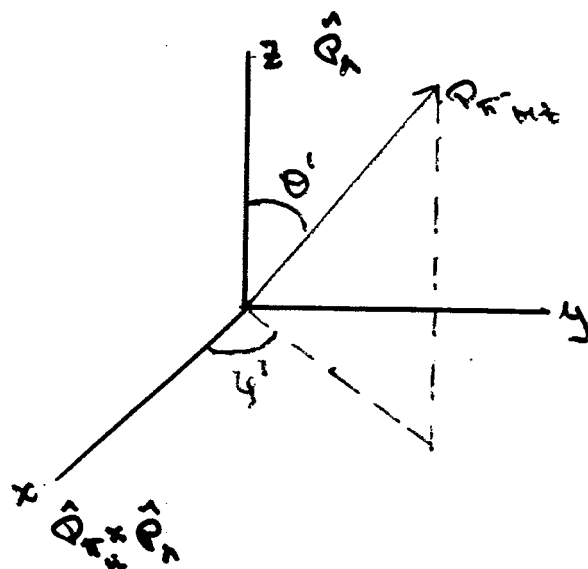
ϕ in prod. plane angle. Parity in strong int.

(Integrated over prod. angle. ϕ).

Alternate Coord. System:

x axis. normal to Prod. plane. $\hat{P}_{\text{Prod}} \times \hat{P}_A$

z axis. \hat{P}_A



θ' Parity in Scat Int. Forward Backward.

ϕ' Parity in Wreal Int. Quad. $\text{I} + \text{IV}$ up
 $\text{II} + \text{III}$ down.

Parity Violation in Weak Int. N^0 decay.

$$I(\theta, w, y) = (1 + \alpha P_A(w) \cos \theta)$$

$$\overline{\cos \theta} = \frac{\int \cos \theta I(\theta)}{\int I(\theta)} = \frac{\alpha \bar{P}}{3}$$

$$\therefore \alpha \bar{P}_A = 3 \frac{\sum \cos \theta_i}{N} \pm \sqrt{\frac{3}{N}}$$

If just use Up, down.

$$\alpha \bar{P}_A = \frac{2(Up - Down)}{Up + Down} \pm \frac{2}{\sqrt{N}}$$

For Unpolarized N^0 's.

Decay Products - longitudinally polarized.

$$\vec{Q}_P = \alpha_A \hat{Q}_P \quad \text{measure } \alpha_A$$

In similar way for Ξ^0 's.

$$I'(\theta) = (1 + \alpha_{\Xi} P_{\Xi}(\cos \theta))$$

$$\theta^* \neq n \cdot \vec{P}_A$$

$$n = \hat{P}_{cm} + \hat{P}_3$$

Results.

Buddel. 22 N's.

 θ' 10/12 ~~Forward~~ Back ϕ' 14/8 ~~Forward~~ ~~Back~~ up/down.

Samples. 41 events.

 θ' 22/19 F/B ϕ' 24/17 U/D. \rightarrow

$$\alpha_{\overline{P}} = .34 \pm .31$$

Purity non-cons. in N's decay.
Steinberger: 263 events

$$\theta \quad \alpha_{\overline{P}} = .40 \pm .11$$

165 Σ^- events

$$\alpha_{\Sigma^- \overline{P}} = -.07 \pm .14$$

Grawford: 353 events

$$\theta \quad \alpha_{\overline{P}} = .44 \pm .11$$

$$\alpha_{\Lambda} = .64$$

$$\alpha_{\Sigma^-} = -.07$$

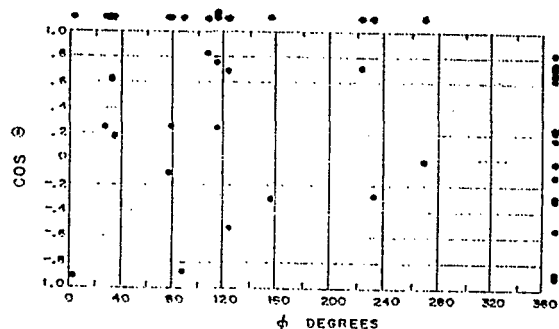


FIG. 8. Angular correlation plot of the cosine of the polar angle against the azimuthal angle ϕ for the reaction $\pi^- + p \rightarrow \Sigma^- + K^+$, $\Sigma^- \rightarrow \pi^- + \pi^0$. The point histograms on the edges of the figure represent the data after integration over the other coordinate.

clearly demonstrated. In fact, the data may be used to check this hypothesis, since any experimentally established violation of the symmetries above must be attributed to a lack of parity conservation.¹¹

In Fig. 11 we show the dihedral angles in the range 0° – 90° , after making use of the above symmetry properties. These may be compared with the results reported by Fowler *et al.* and Walker *et al.*, on the basis of eight Λ^0 events, six θ^0 events, and three Σ^- events. There is no indication of polarization in θ^0 production. However, the earlier results in the case of Λ^0 production in hydrogen show all eight events with dihedral angles between 0° and 45° . Twelve of our 23 events in which a Λ^0 was seen have dihedral angles in the same region. The combination of all these thirty-one Λ^0 , Σ^0 production events in hydrogen gives

$$N(0^\circ\text{--}45^\circ)/N(45^\circ\text{--}90^\circ) = 20/11.$$

We believe that the statistics are not adequate to demonstrate a polarization effect. For the Σ^- , the ratio is

$$N(0^\circ\text{--}45^\circ)/N(45^\circ\text{--}90^\circ) = 10/6;$$

here again statistics are inadequate to establish polarization.

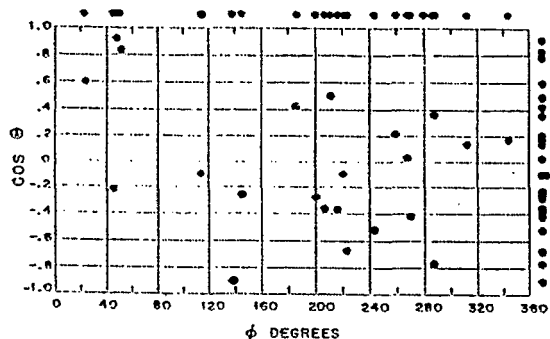


FIG. 9. Angular correlation plot for the reaction $\pi^- + p \rightarrow \Lambda^0 + \theta^0$, $\Lambda^0 \rightarrow \pi^- + \pi^0$.

¹¹ We wish here to acknowledge some very helpful discussions with T. D. Lee on these questions, and a communication from R. Karplus.

Adair¹² has pointed out that for the production angle β near zero or 180° , and assuming the spin of the θ^0 (or K^+) to be zero, the angular distribution in θ of the Λ^0 (or Σ^-) decay is determined by angular momentum and parity conservation alone. For example, for the spin of the hyperon $\frac{1}{2}$, the distribution in θ is isotropic, for spin $\frac{3}{2}$ it is $\frac{3}{2} \cos^2\theta + \frac{1}{2}$, etc.

We have calculated the likelihood functions for these distributions, using those eleven Λ^0 and eight Σ^- events with $|\cos\beta| \geq 0.78$.

$$\text{For the } \Lambda^0: \frac{L_{\frac{3}{2}}(\theta)}{L_{\frac{1}{2}}(\theta)} = \prod_{i=1}^N \frac{(\frac{3}{2} \cos^2\theta_i + \frac{1}{2})}{1} = 0.28.$$

$$\text{For the } \Sigma^-: \frac{L_{\frac{3}{2}}(\theta)}{L_{\frac{1}{2}}(\theta)} = 0.80.$$

For greater assumed spin j , the likelihood functions L_j would become progressively smaller. There is certainly no evidence here that the hyperon spins are larger than $\frac{1}{2}$; the best agreement is obtained with spin $\frac{1}{2}$.

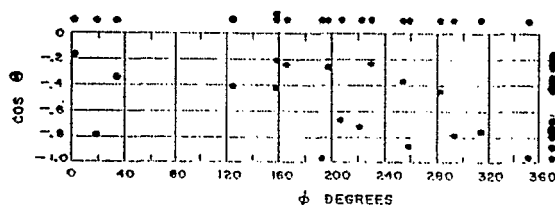


FIG. 10. Angular correlation plot for the reaction $\pi^- + p \rightarrow \Lambda^0 + \theta^0$, $\theta^0 \rightarrow \pi^+ + \pi^-$. The θ variation extends from $\pi/2$ to π since the π^+ and π^- are indistinguishable in this experiment.

for both Λ^0 and Σ^- . The data are still inadequate, but this kind of analysis seems promising.

VII. ANOMALOUS Λ^0 OR θ^0 PARTICLES

It is the underlying hypothesis of this paper, insofar as concerns neutral unstable particle production, that the observation of a single decay establishes the event. The trajectory and momentum of the unobserved particle can then be obtained from the kinematics, and as seen in Table I, the potential path of the unobserved particle has also been measured. The potential path is defined as the length of path the particle would have had, if it had not decayed, minus $\frac{1}{2}$ cm; the latter being approximately the length of decay tracks necessary to detect the decay.

It is then possible to ask: given a Λ^0 (θ^0) which is identified as coming from the process $\pi^- + p \rightarrow \Lambda^0 + \theta^0$, are the number of θ^0 's (Λ^0 's) observed with these Λ^0 's (θ^0 's) compatible with the known potential path lengths and the known lifetimes? We have observed only four double events despite the fact that the average potential path is of the order of the average mean free path both

¹² R. K. Adair, Phys. Rev. 100, 1540 (1955).

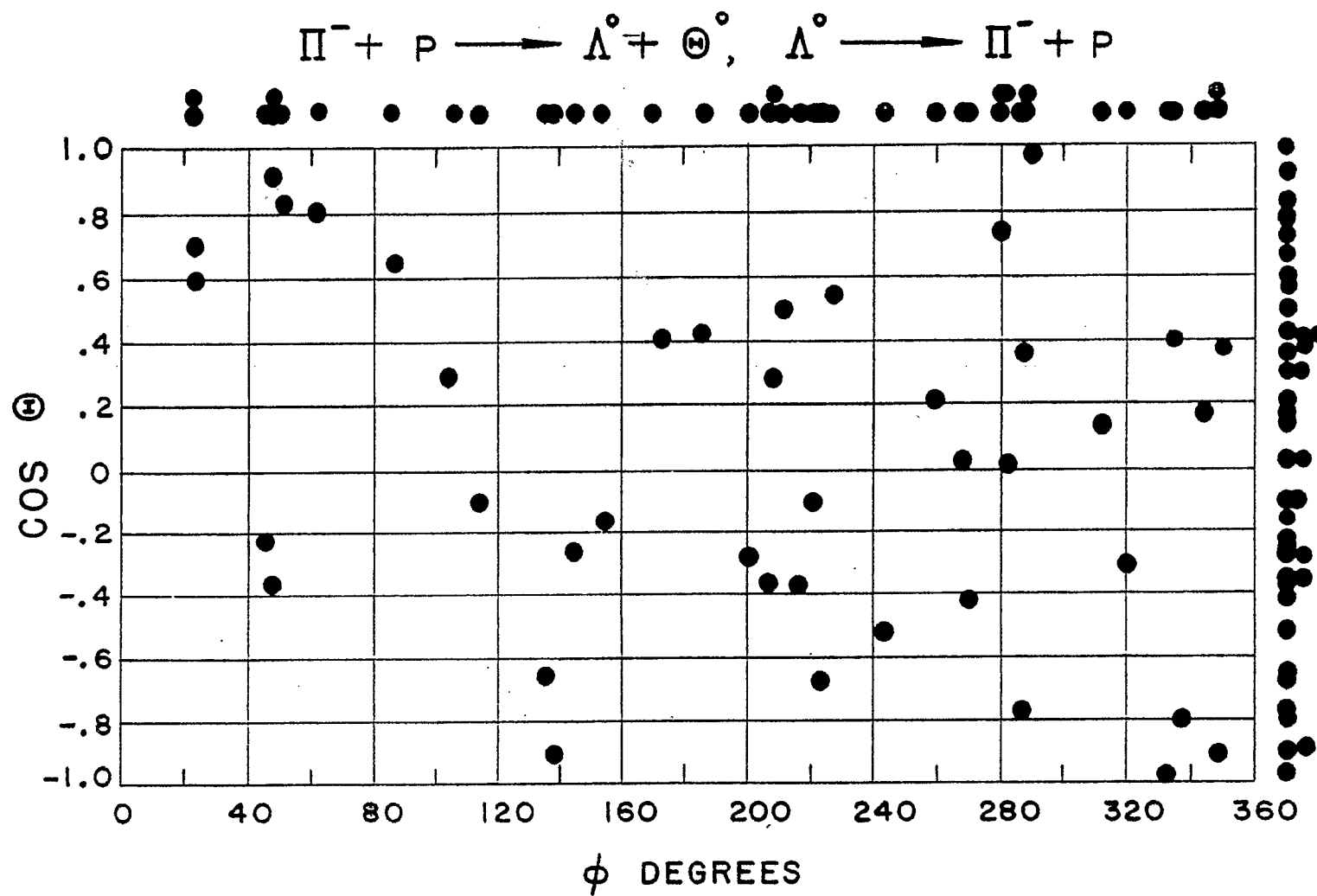
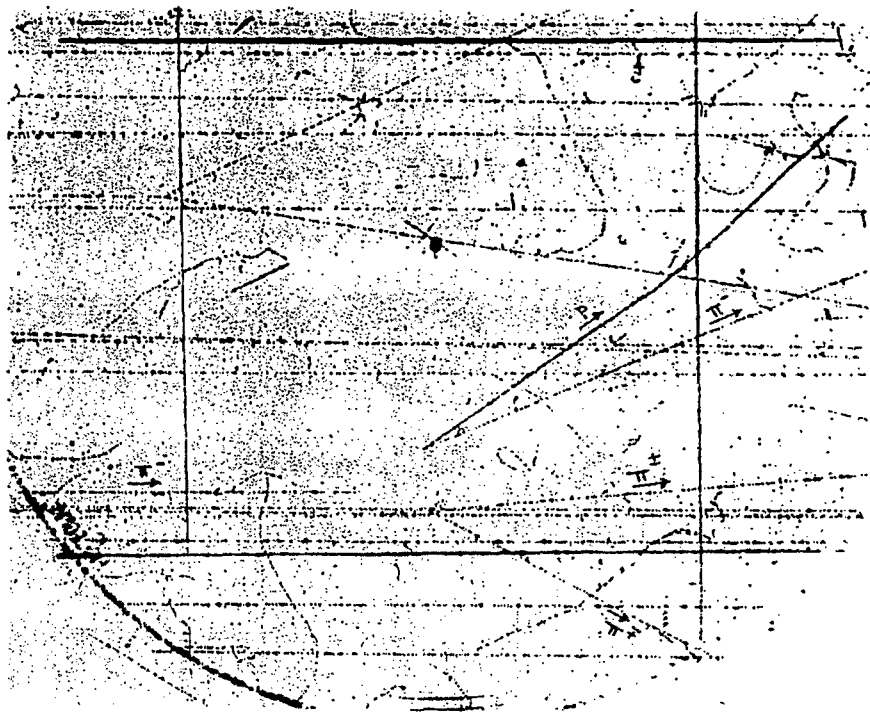


FIG. 12

FIG. 5. Λ^0 - θ^0 production in which both V 's are seen. The Λ^0 shows, characteristically, a heavily ionizing proton and a minimum-ionizing π^- . The θ^0 shows two minimum-ionizing π 's.



quired that the bubble densities, multiple scattering, and the ranges of the secondaries, when available, be consistent with the kinematics of the reaction. In judging the precision of the method, it should be kept in mind that the end of the incident π^- is not precisely defined because of the finite bubble density. The probable error in position due to this effect is $\sim \frac{1}{2}$ mm. The average Λ^0 and θ^0 path length is ~ 2 -3 cm, but those with path lengths of the order of a few mm are not measured with good precision. The usual accuracy in the determination of the energy of the Λ^0 and θ^0 is ~ 5 -15%.

The evidence for the existence of a Σ^0 with a mass near that of the Σ^+ is still not conclusive. However, there are strong theoretical reasons⁶ to postulate this particle and its decay in a very short time ($\sim 10^{-20}$ sec) into a γ ray and Λ^0 with a Q of approximately 70 Mev. In our pictures this production event would appear almost identical to reaction (2) (since the Σ^0 would decay at the origin), but with the following differences:

1. If a θ^0 appears, for a given angle of appearance the kinetic energy would be lower than that in a Λ^0 - θ^0 production event by an amount which varies from $\sim 20\%$ at small production angles to $\sim 50\%$ at larger ones.

2. If a Λ^0 appears, its energy is not uniquely determined by the production angle. It can, in fact, have a reasonably large spread depending upon the angle of emission of the γ ray.

Of the 38 events in categories (1) and (2), only 4 exhibit both V 's in the chamber; 3 of these fit Λ^0 - θ^0 production well, the fourth fits well for Σ^0 - θ^0 production

and does not seem reconcilable with (2). The remaining 34 events consist of only one observed V . Of these, 19 are Λ^0 's and 15 are θ^0 's. In these categories, 15 events cannot be measured well enough to distinguish Λ^0 and Σ^0 production confidently. 17 others are definitely identifiable as Λ^0 - θ^0 production, and only 2 events exist for which Σ^0 production seems definitely indicated. Both the latter are events in which a θ^0 was observed; no events in which a single Λ^0 was seen can be unambiguously interpreted as Σ^0 - θ^0 production.

IV. BEAM ENERGY

The most accurate determination of the beam energy utilizes two events of the type $\pi^- + p \rightarrow \Sigma^- + K^+$ in which the K^+ comes to rest without nuclear interaction. Using mass values $M_{\Sigma^-} = 2344 m_e$ ⁷ and $M_{K^+} = 965.5 m_e$,⁸ we find for the beam momentum in the 2 cases, $P_{\pi} = 1.420 \pm 0.005$ Bev/c and $P_{\pi} = 1.447 \pm 0.005$ Bev/c, respectively; these yield a mean momentum $P_{\pi} = 1.433 \pm 0.015$ Bev/c. The latter error indicates the beam momentum spread determined from the calculated trajectories.

V. PRODUCTION ANGULAR DISTRIBUTIONS

The center-of-mass distributions for the 38 events which are either Λ^0 - θ^0 or Σ^0 - θ^0 production are presented together in Fig. 6. In the same figure the center-of-mass angles of the seventeen $\Sigma^- - K^+$ events are also shown.

⁷ Chupp, Goldhaber, Goldhaber, and Webb, "Proceedings of the International Conference on Elementary Particles, Pisa, 1955," *Nuovo cimento*, (to be published).

⁸ Heckman, Smith, and Barkas, University of California Radiation Laboratory Report UCR1-3156, 1955 (unpublished).

events. The sample for reaction (2) is less than 3% contaminated.

Results on the anisotropy are summarized in Tables II and III, and the θ distributions for the combined energies are shown in Figs. 1 and 2. The anisotropy coefficients $\bar{P}\alpha$ are calculated from the individually observed angles:

$$\bar{P}\alpha = \frac{3}{N} \sum_{i=1}^N \cos\theta_i \pm \left(\frac{3}{N}\right)^{\frac{1}{2}}.$$

\bar{P} is the polarization averaged over the production angles. $\bar{P}\alpha$ positive means π 's emitted preferentially in the direction $\mathbf{p}_\pi \propto \mathbf{p}_\pi \times \mathbf{p}_\pi$.

The results show a very large, statistically well established anisotropy for the Λ^0 , clearly demonstrating parity violation in the decay. For the entire sample $\bar{P}\alpha = -0.40 \pm 0.11$. For the Σ^- decay no statistically significant anisotropy is observed. The conclusions are strengthened by the very similar results obtained by Crawford *et al.*⁶ At a kinetic energy of 0.99 Bev, this group obtains⁷ $\bar{P}\alpha = 0.51 \pm 0.15$ for Λ^0 decay, and also no measurable anisotropy for Σ^- decay.

To estimate the anisotropy coefficient, α , itself, we have combined all results available in the lower energy interval, where the results of Table II indicate a larger polarization. With our results at 910, 950, and 1100

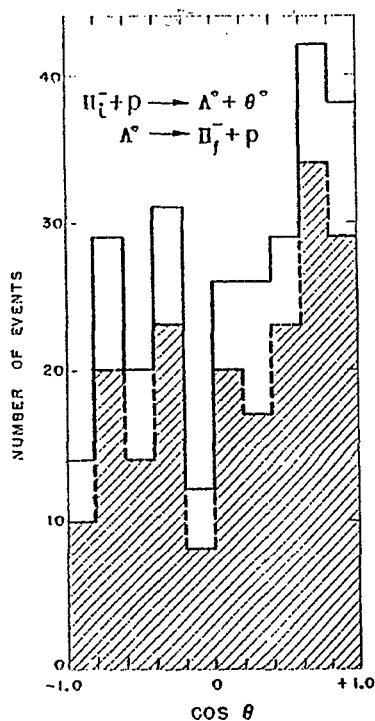


FIG. 1. Distribution in $\cos\theta$ for process (1). The shaded area represents events for production angles in the center-of-mass range 30° - 150° .

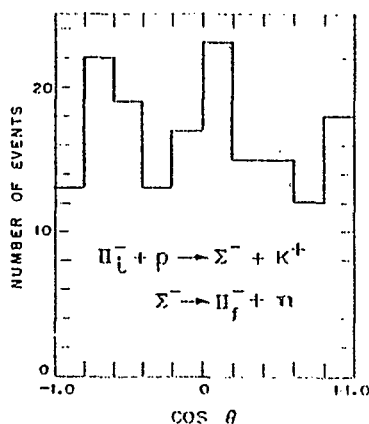


FIG. 2. Distribution in $\cos\theta$ for process (2).

Mev, with those of Berkeley at 990 Mev,⁶ and with the result $\bar{P}\alpha = 0.465 \pm 0.34$ of Adair and Leipuner at 950 Mev,⁸ one obtains $\bar{P}\alpha = 0.52 \pm 0.10$. To find α it is necessary to know P . This is not possible at present; however, it is possible to fix an upper limit for $|\bar{P}|$ from the observed angular distributions.⁹ In this energy region only S and P waves can contribute appreciably to the angular distribution. The cross section then has the form $|a + b \cos\theta|^2 + |c|^2 \sin^2\theta$ and $\bar{P} = (2\pi^2/\sigma) \text{Im}ac^*$. If we then use the fact that the same group of data gives a backward to forward asymmetry for the production of Λ^0 's in process (1) of 2.9 ± 0.4 , we obtain an upper limit for $|\bar{P}|$: $|\bar{P}| \leq 0.78 \pm 0.03$. This results in a lower bound on $|\alpha|$: $|\alpha| \geq 0.67 \pm 0.13$.

* Results presented at the Venice Conference on Elementary Particles, September 22-28, 1957 (unpublished).

† This research is supported by the U. S. Atomic Energy Commission and the Office of Naval Research.

¹ Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957).

² Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415 (1957).

³ T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).

⁴ L. Landau, Nuclear Phys. 3, 127 (1957); T. D. Lee and C. N. Yang, Phys. Rev. 105, 1671 (1957); A. Salam, Nuovo cimento 5, 299 (1957).

⁵ Lee, Steinberger, Feinberg, Kabir, and Yang, Phys. Rev. 106, 1367 (1957).

⁶ Crawford, Cresti, Good, Gottstein, Lyman, Solnitz, Stevenson, and Ticho, Phys. Rev. 108, 1102 (1957). We wish to thank these authors for an advance copy of their results.

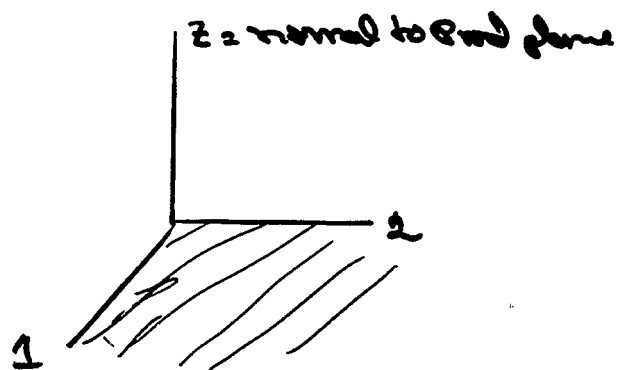
⁷ We have taken the liberty to treat these results somewhat differently than the authors have, and obtain a somewhat altered result. The Λ^0 data consist of 76 clear events (double decays) which give $(3/N)\Sigma \cos\theta_i = 0.367 \pm 0.20$. In addition, there are 277 single Λ^0 events of which, according to the authors, about 15% are θ^0 contaminations, and of the remainder $\sim 25\%$ are Σ^0 contaminations. Since neither contaminant should contribute to the anisotropy, the up-down asymmetry of 0.57 (only the sign of θ has been determined for these events) refers to an effective sample of 177 events: $\bar{P}\alpha = 2 \times (167 - 110)/177 = 0.644 \pm 0.20$. The combined result is $\bar{P}\alpha = +0.51 \pm 0.15$. The result obtained by the authors, using only up and down results and neglecting the correlation for contamination, is 0.44 ± 0.11 .

⁸ R. Adair and L. Leipuner (to be published). We wish to thank the authors for making these results available to us.

⁹ Report of this group at the Venice Conference on Elementary Particles, September, 1957 (unpublished).

Parity Violation in Strong Interaction.

Polarization in Production Plane.



$$\alpha P_1 = \frac{3}{N} \sum_i \cos \theta'_i \quad \theta' \neq \hat{p}_{\text{beam}} \cdot \hat{1} \quad \text{Front/Back}$$

$$\alpha P_2 = \frac{3}{N} \sum_i \cos \theta''_i \quad \theta'' \neq \hat{p}_{\text{beam}} \cdot \hat{2} \quad \text{Left/Right.}$$

$$(\alpha P_2 = \frac{3}{N} \sum_i \cos \theta_i) \text{ Parity asym. cons. weak int.}$$

Crowl et al. $236 \text{ events} \cdot 1.126 \text{ u/c}$
 $185 \text{ events} \cdot 1.236 \text{ u/c}$

$$\alpha P_1, \alpha P_2 \quad \begin{matrix} 0 \pm .11 \\ 0 \pm .13 \end{matrix} \quad \begin{matrix} .08 \\ .08 \end{matrix} \quad \alpha P_2 = .66 \pm .13$$

$$P < \frac{.13}{.64} < .20 \quad \frac{.08}{.64} = .10$$

$$\Delta P = \frac{1}{2} \sqrt{\frac{3}{N}} = \frac{271}{\sqrt{N}} \quad \sim 70,000 \text{ events for } 1\% \Delta P.$$

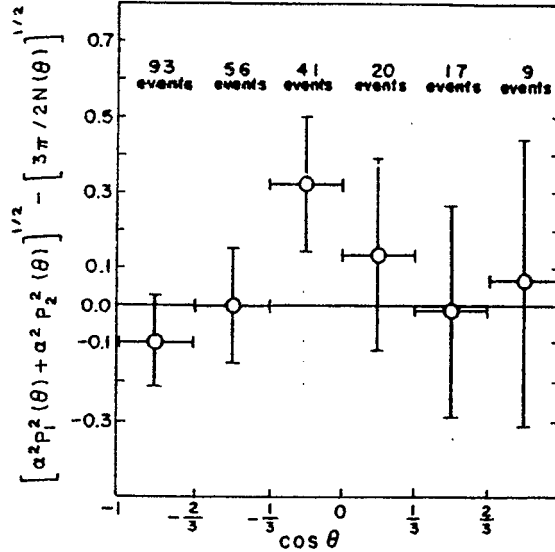


FIG. 1. Magnitude of the Λ decay asymmetry in the production plane, minus $[3\pi/2N(\theta)]^{1/2}$, the mean value expected from statistical fluctuations alone, plotted versus θ , the hyperon c.m. production angle. The plotted errors are the rms fluctuations $\pm[(2-\pi/2)3/N(\theta)]^{1/2}$.

tems might be expected to be "preferred" in the sense that the polarization in the production plane would not cancel vectorially in averaging over θ .

The results are summarized in Table I. No statistically significant average polarization in the production plane is apparent in any of the coordinate systems.¹¹ (This result was, of course, guaranteed by the negative result from the preceding "coordinate-invariant" analysis.)

We now adopt the hypothesis that parity is not conserved in production in order to determine an upper limit to the parity-nonconserving amplitude. In the notation of Drell *et al.*⁵ and Lee *et al.*,⁸ the production matrix element (M.E.)

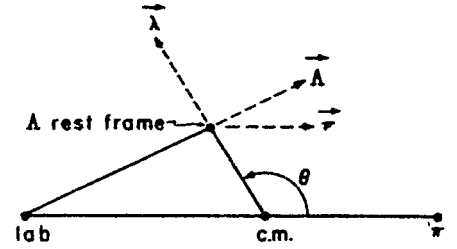


FIG. 2. Mnemonic (nonrelativistic) diagram in velocity space. $\vec{\pi}$, $\vec{\Lambda}$, and $\vec{\lambda}$ are unit vectors, referring to the direction of the incident π with respect to the center of mass, the Λ with regard to the laboratory frame, and the Λ with regard to the c.m., all as seen in the Λ rest frame. See also reference 9.

may be written

$$M.E. = a + b \cos \theta + i c \sin \theta \vec{\sigma} \cdot \vec{n} + d \vec{\sigma} \cdot \vec{\pi},$$

where d is the parity-nonconserving amplitude. Then, in the " π -c.m." coordinate system (Fig. 2 and Table I), we have

$$I(\theta)P_n(\theta) = 2 \text{Im} c^* (a + b \cos \theta) \sin \theta,$$

$$I(\theta)P_\pi(\theta) = 2 \text{Re} d^* (a + b \cos \theta),$$

$$I(\theta)P_2(\theta) = 2 \text{Re} d^* c \sin \theta,$$

$$I(\theta) = |a + b \cos \theta|^2 + |c \sin \theta|^2 + |d|^2.$$

After averaging over θ we have

$$\langle IP_n \rangle = (\pi/2) \text{Im} c^* a, \quad (3)$$

$$\langle IP_\pi \rangle = 2 \text{Re} d^* a, \quad (4)$$

$$\langle IP_2 \rangle = (\pi/2) \text{Re} d^* c, \quad (5)$$

$$I = |a|^2 + |b|^2/3 + 2|c|^2/3. \quad (6)$$

Since P_n is observed to be large, c and a must both be nonzero, and their phase difference cannot be 0° or 180° . Therefore, P_π and P_2 cannot both vanish, unless $|d|$ is 0.

If we eliminate the phase of d from Eqs. (3),

Table I. Polarization components averaged over hyperon c.m. production angle. Here \vec{n} is a unit vector in the direction $\vec{P}(\pi \text{ incident}) \times \vec{P}(\text{hyperon})$. The standard deviations on all $\alpha \bar{P}_{1,2,3}$ are $(3/236)^{1/2} = 0.113$. Prob. $(\chi_{1,2}^2) = \exp\{-[\alpha^2 \bar{P}_1^2 + \alpha^2 \bar{P}_2^2]/236/6\}$ = the probability of getting a χ^2 as large as or larger than that observed, if the true values are $\bar{P}_1 = \bar{P}_2 = 0$.

Coord. system	Axis No. 3	Axis No. 1	Axis No. 2	$\alpha \bar{P}_3$	$\alpha \bar{P}_1$	$\alpha \bar{P}_2$	Prob. $(\chi_{1,2}^2)$
π -c.m.	\vec{n}	$\vec{\pi}$	$\vec{n} \times \vec{\pi}$	0.55	-0.13	+0.15	0.21
Λ -c.m.	\vec{n}	$\vec{\lambda}$	$\vec{n} \times \vec{\lambda}$	0.55	+0.087	+0.068	0.62
Λ -lab	\vec{n}	$\vec{\Lambda}$	$\vec{n} \times \vec{\Lambda}$	0.55	-0.046	+0.18	0.26

decay to the first excited state of Na^{23} is predominantly G-T since, if one accepts the Fermi transition as vector,¹ any admixture of a Fermi transition would lead to a less negative value of λ than $-\frac{1}{3}$.

*Work performed under the auspices of the U. S. Atomic Energy Commission and also supported in part by the Office of Naval Research.

[†]Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

¹B. W. Ridley, *Nuclear Phys.* **6**, 34 (1958).

²Herrmannsfeldt, Maxson, Stähelin, and Allen, *Phys. Rev.* **107**, 641 (1957).

³Herrmannsfeldt, Stähelin, and Allen, *Bull. Am. Phys. Soc. Ser. II*, **3**, 52 (1958).

⁴Herrmannsfeldt, Burman, Stähelin, Allen, and Raid, *Phys. Rev. Lett.* **1**, 61 (1958).

⁵J. R. Penning and F. H. Schmidt, *Phys. Rev.* **105**, 17 (1957).

FURTHER SEARCH FOR PARITY NONCONSERVATION IN ASSOCIATED PRODUCTION*

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Myron L. Good, Frank T. Solmitz, and
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(Received December 8, 1958)

Several authors have pointed out that the case for parity conservation in strong interactions is very much weakened when strange particles are involved.^{1,2} If parity is not conserved in the associated production process

$$\pi^- + p \rightarrow \Lambda + K^0, \quad (1)$$

then the Λ may have a polarization component in the production plane. The parity-nonconserving decay

$$\Lambda \rightarrow p + \pi^- \quad (2)$$

may then, by virtue of its large decay-asymmetry parameter, exhibit a decay asymmetry in the production plane.

In an earlier Letter we reported our analysis of 236 events of the type (1)+(2), produced by 12-Bev/c pions incident upon a liquid hydrogen bubble chamber, leading to Λ 's of 300 Mev/c m. momentum.³ Those results were consistent with zero decay asymmetry in the production

plane. We now report our analysis of 185 events of the same type, but produced at a higher energy by pions of 1.23 Bev/c, leading to 375-Mev/c Λ 's in the c.m. system.

One might expect from statistical considerations that adding 185 events to an existing 236 could hardly change the conclusions. However, (a) it turns out that, partly because of a larger observed up-down decay asymmetry, and partly because of a smaller observed decay asymmetry in the production plane, we can set a substantially smaller limit (about one-third as large) to the amount of parity-nonconserving amplitude in the experiment reported here than in the 1.12-Bev/c experiment; and (b) it is conceivable that a parity-nonconserving production amplitude could increase substantially between 300 and 375 Mev/c.

Figure 1 shows the observed decay-asymmetry components in the production plane plotted against θ , the hyperon c.m. production angle. In the left half of the figure we plot the front-back (FB) asymmetry in the π -c.m. coordinate system, in which the positive direction is along \vec{P} (π incident). The right half of the figure shows the left-right (LR) asymmetry in the same system. The positive direction is along $\vec{n} \times \vec{P}$ (π inc), where \vec{n} is the "up" direction given by $\vec{P}(\pi \text{ inc}) \times \vec{P}(\text{hyperon})$. All directions are as seen in the hyperon rest frame. These data are clearly consistent with zero asymmetry. A χ^2 test applied to the hypothesis that the FB asymmetry is everywhere zero yields $\chi^2(\text{FB}) = 7.3$, where 6 is "expected." Similarly $\chi^2(\text{LR}) = 1.0$. These combine to give a total $\chi^2 = 8.3$, where 12 is expected if the asymmetry is identically zero. This corresponds to a χ^2 probability of 76%.

The contribution to χ^2 at each value of θ is just the square of the magnitude of the projection of

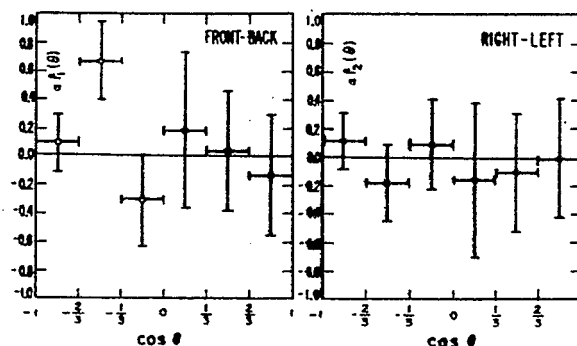


FIG. 1. Decay asymmetry components in the production plane (see text).

Resume:

$$\pi^- p \rightarrow \Lambda + 0$$

$$\hookrightarrow \pi^- p. \quad d_{\Lambda} = .64$$

Apply to $Au \times Au. \rightarrow \Lambda^0 \text{'s} + \dots$

Normal to Prod. Plane. $\vec{P}_{Au} \times \vec{P}_{\Lambda}$

If Parity Violated in Strong Int.

Λ^0 polarization in Production Plane.

Concerns:

$$\Sigma^0 \rightarrow \Lambda^0 \gamma$$

$$\Xi^- \rightarrow \Lambda^0 \pi^-$$

$$d_{\Xi} = -.46$$

$$\Xi^0 \rightarrow \Lambda^0 \pi^0$$

Systematics - acceptance --

Precision - harmonic

$$\phi_{\text{deg}} = \mu_{\Lambda} \int \frac{B d\theta}{\mu}$$

$$\approx 18^\circ \text{ for } \int B d\theta \text{ 1 Total Meke.}$$

AGS Exp's. - Producing Λ^0 's.
Observing

E-810 MPS-TPC. Si-Pb. 40 cm
 Au x Au 80 cm.

Central Events

Reproduce Λ^0 's, K^0 's, Ξ^- 's.

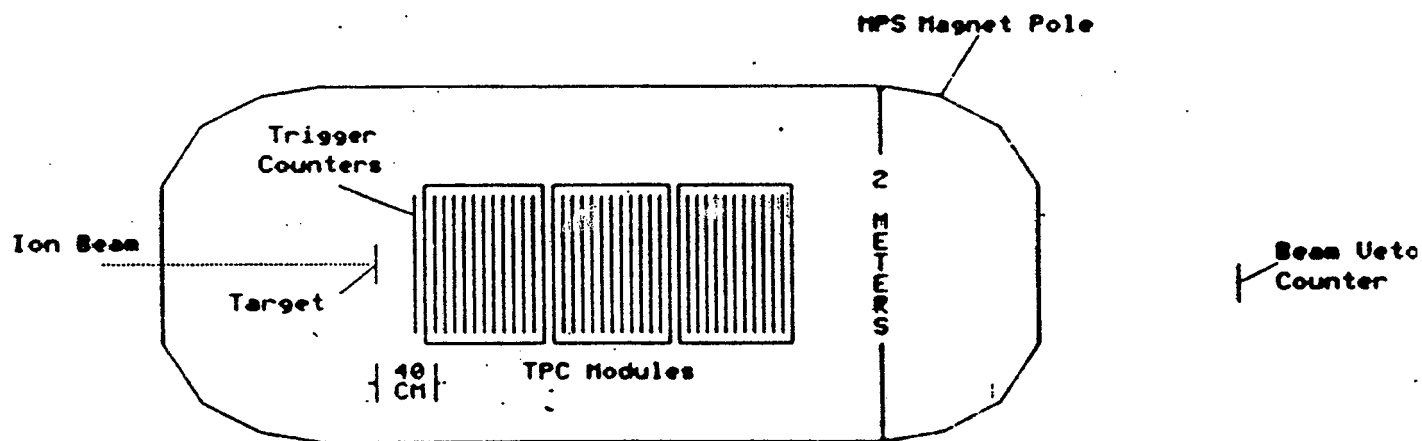
Si-Si. $\sim 1000 \Lambda^0$'s. $1.4 < y < 3.2$

Si-Pb $\sim 3,000 \Lambda^0$'s. "

Si-Pb $\sim 100 \Xi^-$'s. $1.4 < y < 2.9$

E-891

Au x Au. $\sim 2700 \Lambda^0$'s. $2.2 < y < 3.2$



E-810 Plan View

Figure 1

restriction on conventional cascade model calculations which predict the distributions of all particles produced in heavy ion interactions.

2. Experimental method

The experimental method was described in previous publications^{5,6}. Briefly, experiment E810 measured charged tracks in three TPC (Time Projection Chamber) modules in a magnetic field. The detector covered the forward hemisphere in the center-of-mass. The trigger, as described in Ref. 5, selected centrally enriched events for data recording. For the final data sample we selected the most central events using a cut on the highest multiplicity of the negatively charged tracks within our good acceptance. We found this to be a reasonably good measure of the centrality from both the increased yield of K_S^0 's and Λ 's as a function of this multiplicity⁵ and from Monte Carlo studies of the correlation of impact parameter with this multiplicity. We selected the most central events from the *Si* target corresponding to a cross section of approximately 100 mb, and for *Pb* corresponding to a cross section of approximately 300 mb. These cuts correspond to approximately 10% of the geometric cross section. Since we have shown in Ref. 5 that the yield of Λ 's and K_S^0 's is linearly dependent on our centrality selection criterion (negative multiplicity), any tighter cut for centrality selection is not justified because of limited statistics and our estimated systematic error of 20%. Thin targets were used to reduce γ ray conversion which would give incorrect hadron multiplicities. We used a 0.122 cm thick *Si* target (1.3% radiation length) and a 0.02 cm thick *Pb* target (3.5% radiation length). For more details on the experimental method see Refs. 5 and 6. The effective masses for K_S^0 's and Λ 's were calculated by kinematic hypothesis by assigning a proton or a pion mass to the charged tracks which form a vertex away from the point of interaction (see Ref. 6 for more details).

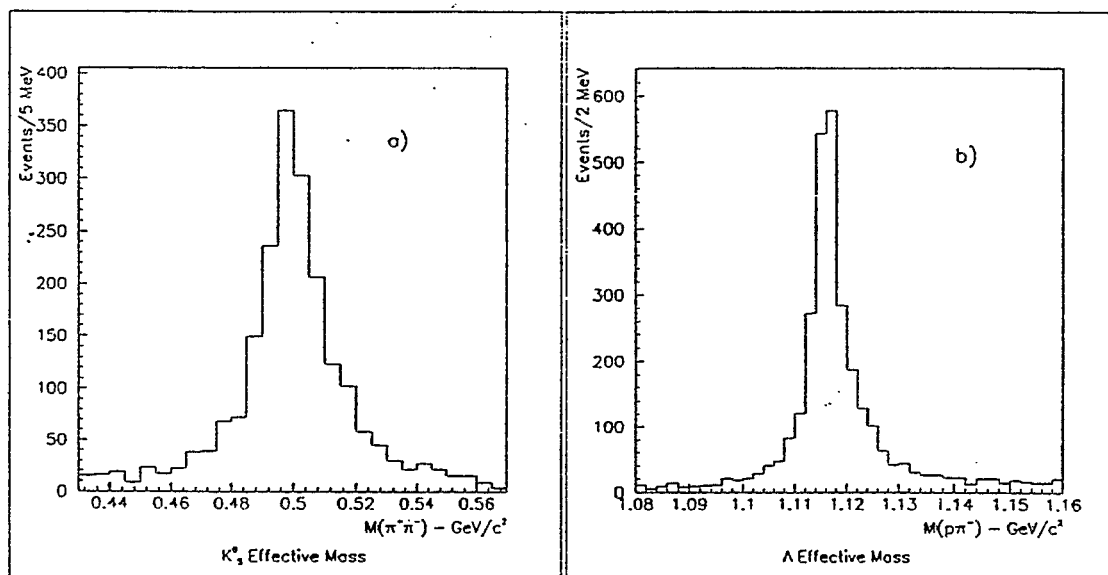
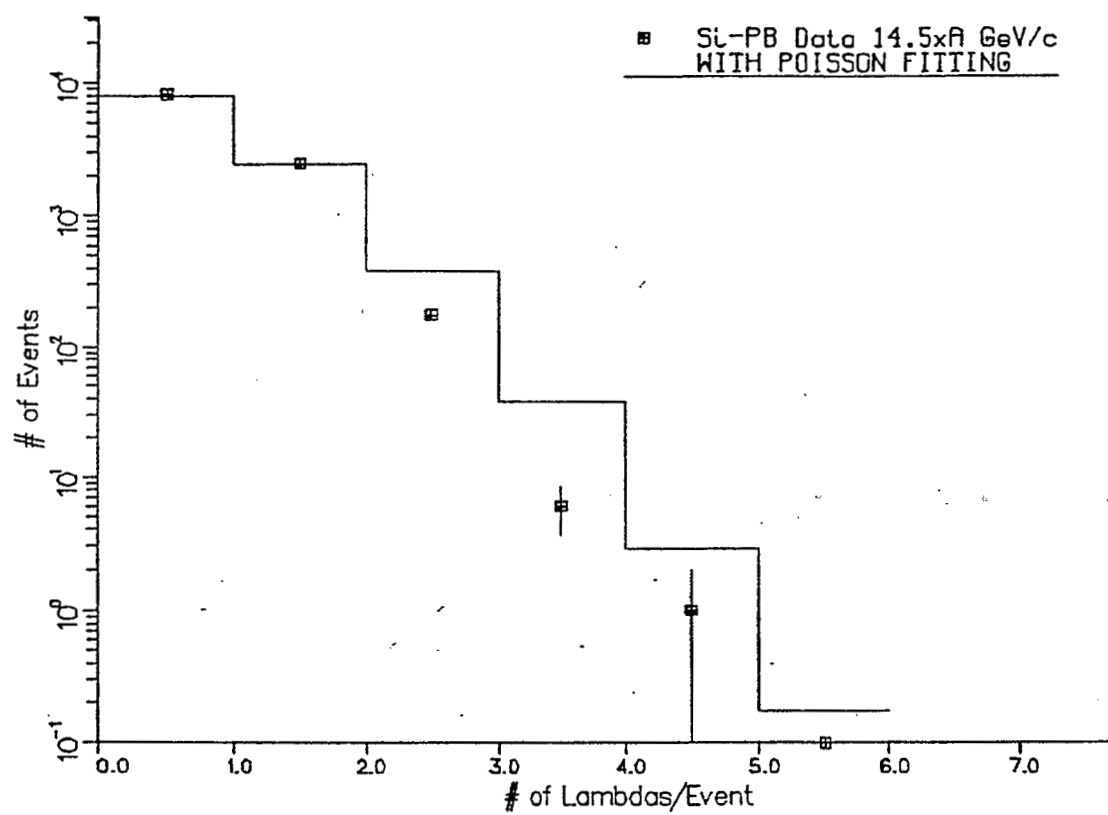


Fig. 1: (a). Effective mass plot of the $\pi^+\pi^-$ hypothesis for decay vertices from *Pb* target with vertices removed if they satisfy the Λ effective mass cuts. (b) Effective mass plot of the proton π^- hypothesis for decay vertices from *Pb* target.

Lambda Multiplicity



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Figure[5.25] Number of central events with multi- Λ production.

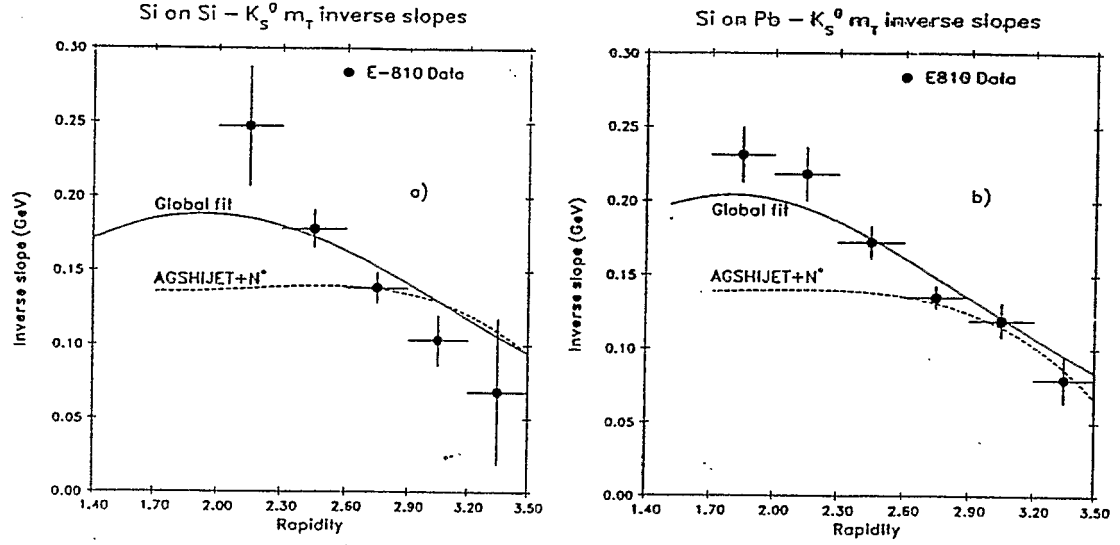


Fig. 3: (a) Inverse exponential slopes for K_s^0 's from the *Si* target. The points are fits to an exponential in each rapidity bin. The solid curve is the result of our global fit, not a fit to the points. The statistical error on the curve representation is similar to that shown on the individual points. The dashed curve is the prediction of AGSHIJET+N*. (b) Inverse exponential slopes for K_s^0 's from the *Pb* target.

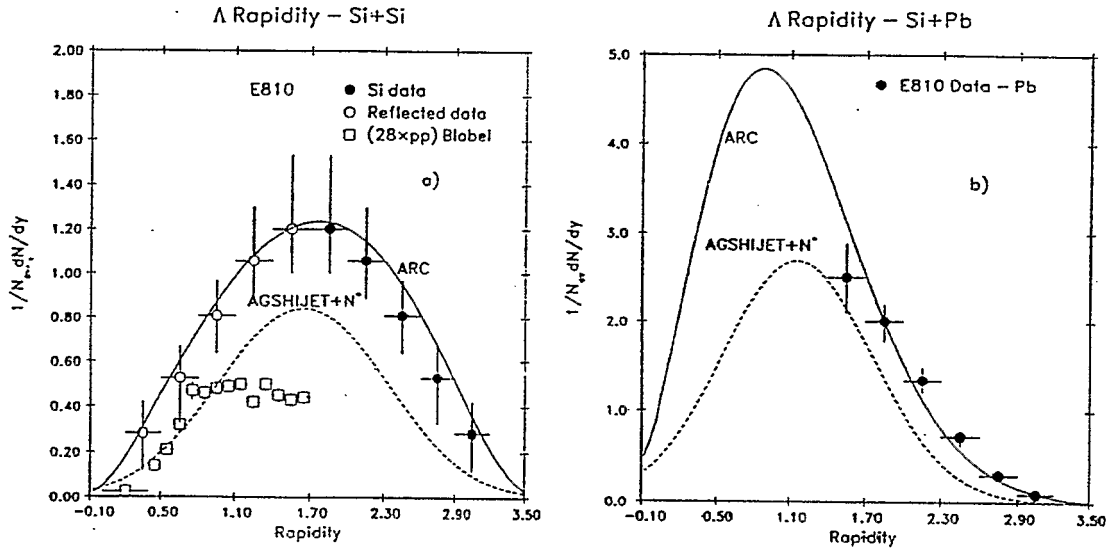


Fig. 4: (a) Rapidity distribution for Λ 's from the *Si* target. The solid points above a rapidity of 1.7 are our measurements. Errors shown are statistical only. The open squares represent the measurements of Ref. 8 scaled up by 28. The solid curve is the prediction of the ARC model. The dashed curve is the prediction of the AGSHIJET+N* model. (b) Rapidity distribution for Λ 's from the *Pb* target.

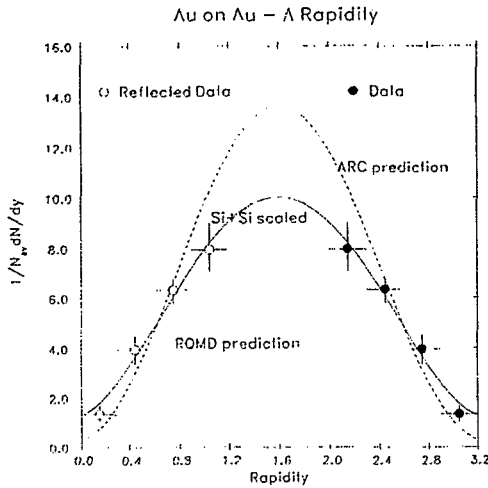


Fig. 4. Rapidity distribution for Λ 's. The solid curve is the result of scaling Si + Si data as explained in the text. The dashed curve is the ARC prediction. The dotted curve is the RQMD prediction. Errors shown are explained in the text.

ward hemisphere about the mid-rapidity point $y = 1.6$. In order to determine the rapidity distributions by integrating over m_t and to determine the contribution due to unmeasured regions of the transverse momentum p_t we have fitted our acceptance corrected data to $A \cdot \exp(-B \cdot m_t)$. Since the coefficients A and B have a strong correlation we have calculated errors using a technique previously described in Ref. [2]. This technique consisted of fitting all of our data points in y and m_t with $A \cdot \exp(-B \cdot m_t)$ where A is an arbitrary constant independent of rapidity and $B = a + b \cdot \cosh(y - y_0) + c \cdot (y - y_0)^2 / m_t$, with a , b and c independent of rapidity and $y_0 = 1.6$. We needed to add the c term to an earlier [2] parametrization in order to obtain a good fit to our data (χ^2 of 38 for 37 D.F.). Without this term we obtain a χ^2 of 94 for 38 D.F. ($CL < 10^{-4}$). To determine the statistical errors we mapped a χ^2 contour of 1σ away from our best fit. It should be noted that these errors contain the contribution due to the uncertainty of extrapolating to unmeasured regions of m_t . We estimate an additional systematic error of $\sim 10\%$. Using the global fit to Λ and K_s^0 data from Si + Si interactions [2] we obtained the ratio of Λ/K_s^0 production. We then used the data of Ref. [7] for K^+ and K^- production on Au + Au and the approximation that $K_s^0 = (K^+ + K^-)/2$ to scale the Λ production of Ref. [2] from Si + Si to

Au + Au. The result of this scaling is shown as a solid curve in Fig. 4. The dashed curve is the prediction of the ARC model and the dotted curve is the prediction of the RQMD model.

5. Discussion and conclusions

As can be seen from Fig. 4, our measurements of the Λ rapidity are in good agreement with the scaling of the rapidity measurements of Ref. [2] by the measured kaon yield from Ref. [7]. In the rapidity region of $2.0 < y < 3.2$, where we have good acceptance for Λ production, we are in good agreement with the predictions of the ARC model for the rapidity distribution. The RQMD model seems to underpredict the yields in that rapidity region. Our measurements of the m_t distributions do not agree near mid-rapidity with the model predictions of either ARC or RQMD as can be seen in Figs. 2 and 3. The m_t inverse slope measurements seem to have a larger increase towards mid-rapidity ($y = 1.6$). This is verified by calculating the χ^2 for our 41 measurements of $1/N \cdot d^2N/dy d\eta_t^2$ in comparison with the two models and the Si + Si measurements of Ref. [2]. For ARC we obtain a χ^2 of 416 for 41 points and a χ^2 of 208 if we allow the overall normalization of the data to change by 30%. For RQMD we obtain a χ^2 of 175 and a χ^2 of 174 for a change of normalization of 5%.

In summary, our measured Λ rapidity distribution for Au + Au central interactions agrees with the scaling of Si + Si measurements of Ref. [2] and with the predictions of the ARC model. The transverse mass distributions become less steep at mid-rapidity compared to the Si + Si measurements or the predictions of ARC and RQMD models. A possible explanation of this effect is increased transverse flow [11] at mid-rapidity in the heavier system.

Acknowledgement

We wish to thank the members of the AGS department and the MPS staff for their support during this experiment. We thank D. Kahana for providing us with events from the ARC model. We are grateful to H. Sorge for access to the RQMD code and advice on running it.

or negatively charged tracks which form a vertex. For more details on Λ reconstruction and its rapidity distribution, please refer to Ref. 7.

With ~ 3000 well defined Λ 's from the Pb target, we successfully found the Ξ^- signal with the following procedures:

We only used those Λ 's and negative tracks (with sagittas ≥ 0.375 cm) that did not come from the primary vertex. When a Λ and a negative track formed a vertex, we took this as a possible Ξ^- decay vertex and extrapolated it to the primary vertex as a helix with a momentum vector of $\vec{P}(\Xi^-) = \vec{P}(\Lambda) + \vec{P}(\pi^-)$. A typical, reconstructed Ξ^- decaying in our TPC module is shown in Fig.1.

For all the vertices that survived the above cuts, we calculated their effective mass with the $\pi^- \Lambda$ hypothesis as plotted in Fig.2a. We selected those 97 candidates that lie in the range of 1.306-1.336 GeV/c^2 as our Ξ^- signal. Those 19 which lie in the range of 1.280-1.294 GeV/c^2 and 1.348-1.364 GeV/c^2 are treated as backgrounds.

3. Results and Discussions

Due to the limited statistics, we can only do a model dependent acceptance correction to our Ξ^- data. In order to calculate acceptances, a complete Monte Carlo simulation of events was performed using GEANT. Events were generated using the AGSHIJET+N* model. The generated TPC's hits included all the known effects of the detectors apertures, efficiencies, resolutions, and distortions. The same code has been used to calculate the acceptance of Λ data and proved successful⁷. The lifetime of the Ξ^- is $c\tau = 4.92$ cm as given by Particle Data Group. We measured the acceptance corrected decay distribution as a function of proper time as shown in Fig.2b, which is in good agreement with the known value. This gives us confidence in our acceptance calculations. The acceptance corrected rapidity spectrum using the AGSHIJET+N* model for the Ξ^- is shown in Fig.3a along with AGSHIJET+N*'s prediction scaled up by a factor of 4. This production is equal to 0.15 Ξ^- per central event

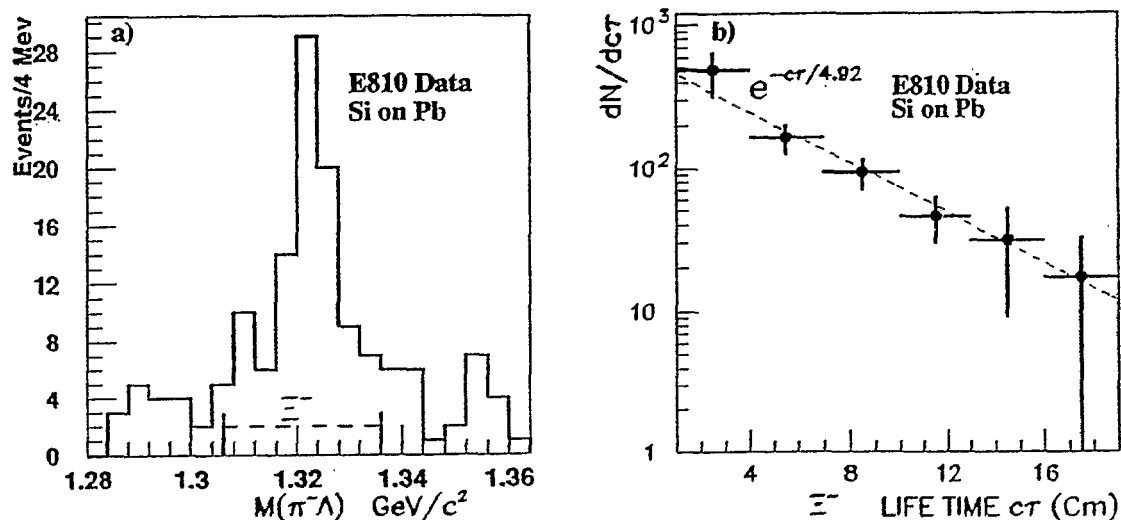


Fig.2 a): Effective mass plot of $\pi^- \Lambda$ hypothesis for the decay vertices. b): The Ξ^- decay distribution from central events. The dashed curve is not a fit, but the known value $e^{-c\tau/4.92}$.

sonably well for $Si+Pb$, although it failed by almost a factor of 2 for $Si+Si$.^{6,7} As pointed out in Ref.12, the enhancement of particles with single strangeness can not serve as a clean signature of the QGP formation, since it carries too much background information of the hadronized state. However, the theory of strangeness enhancement as a signature of QGP still may be important; the question is how to observe it! In a hadron gas, producing multi-strange hyperons requires rescattering among strange hadrons or multiple rescattering of resonant states. This makes their enhancement difficult for the conventional models to account for. During the QGP (Quark Gluon Plasma) phase transition into a HG (Hadron Gas) phase, Ref. 13 demonstrates that a large antistrangeness content will build up in the HG phase while a large strangeness excess will be left in the QGP phase. This excess during hadronization could favor multi-strange hyperon production as well as strangelet formation. With strangelet searches still not successful, we consider hyperons of multiple strangeness a much better probe for QGP than single strangeness searches. To push the strangeness enhancement study to a new stage of QGP search, we have searched for a Ξ^- signal in our data.

2. Experimental Method

E810 was designed to cover a large rapidity range and record as much information as possible on an event by event basis. The detailed experimental method of E810 has been described in previous publications^{5,9}. Briefly, we measured charged tracks in three TPC (Time Projection Chamber) modules in a magnetic field. The detector covered the forward hemisphere in the center-of-mass of the nucleon-nucleon system. The trigger, as described in Ref. 5, selected centrally enriched events for data recording. For the final data sample we selected the most central events using a cut on the highest multiplicity of the negatively charged tracks within our good acceptance. We selected the most central events from the Pb target corresponding to a cross section of approximately 300 mb. These cuts correspond to approximately 10% of the geometric cross section. The effective masses for Λ 's were calculated by kinematic hypothesis by assigning a proton or a pion mass to the positively

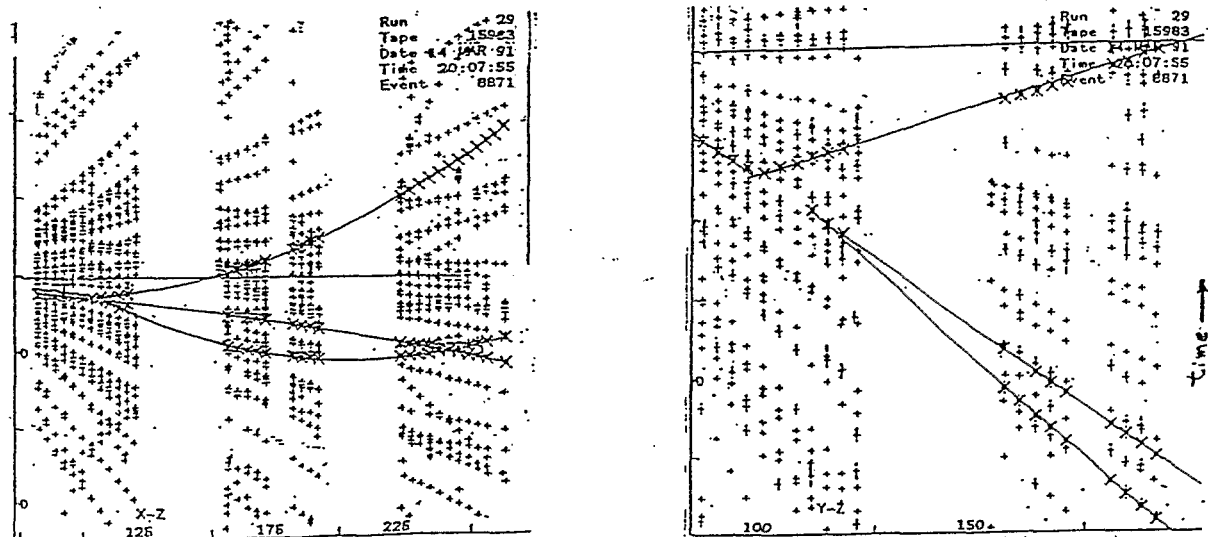


Fig.1 a): The X-Z view of a Ξ^- decaying in TPC module b): Enlarged Y-Z view of the same decaying Ξ^-

E-895

EOS TPC in MPS Magnet.

+ neutron detector + ...

TPC 150 cm x 100 cm x 75 cm

~ 13 kg gas filled.

Au x (Au, Cu, Pb) 2, 4, 6, 8 GeV/A.

~ 2×10^3 ions/pulseRun \approx 800 hours.

Target - close to chamber - 10-20 cm.

E-896

Distributed Drift Chamber. (DDC)

100 cm x 25 cm x 20 cm

1.85 T field

+ neutron detector + ...

Sweeping Magnet.

5 T, 100 cm.

STU.Acceptance for Λ^0 's. ~ 1%.

Expected

$$\text{SDD} \begin{cases} \Lambda^0\text{'s.} & 3,000 \\ \Xi^- & 10 \end{cases}$$

$$\text{CDD} \begin{cases} \Lambda^0\Lambda^0\text{'s.} & \sim 1,000 \\ \Lambda^0 & 10^4 \\ \Xi^- & 10^3 \end{cases}$$
for 10^6 ions/pulse
1,000 ions.

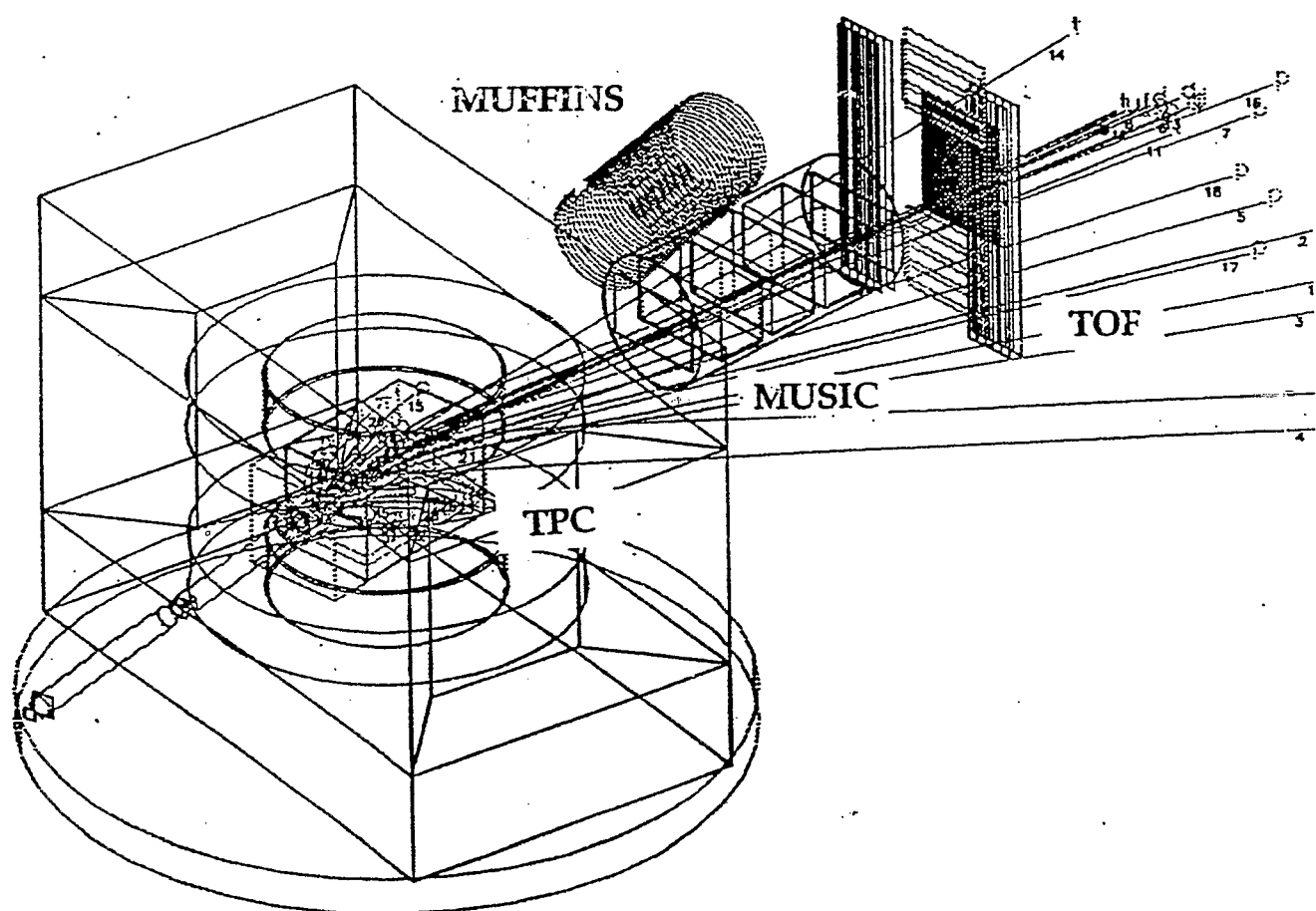


Figure 21: Proposed set up for the EOS experiment at the AGS (perspective view).

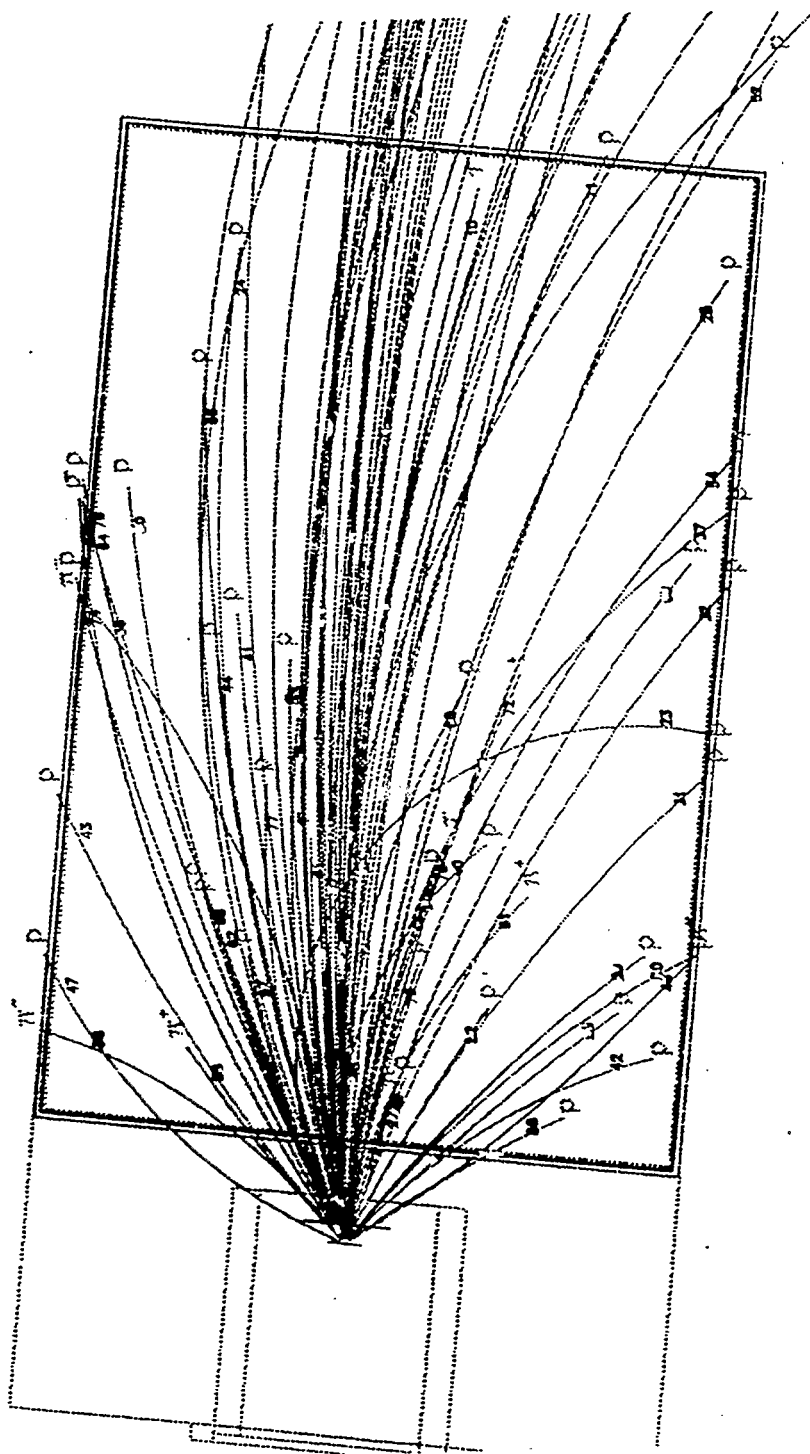
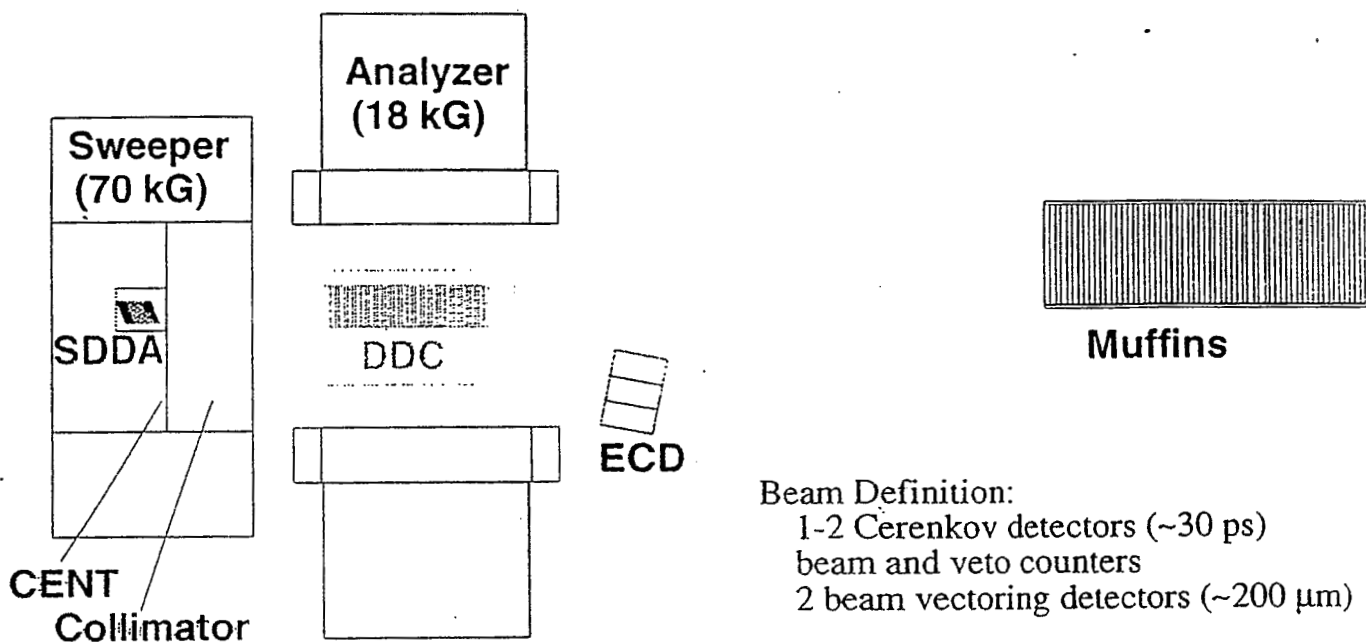
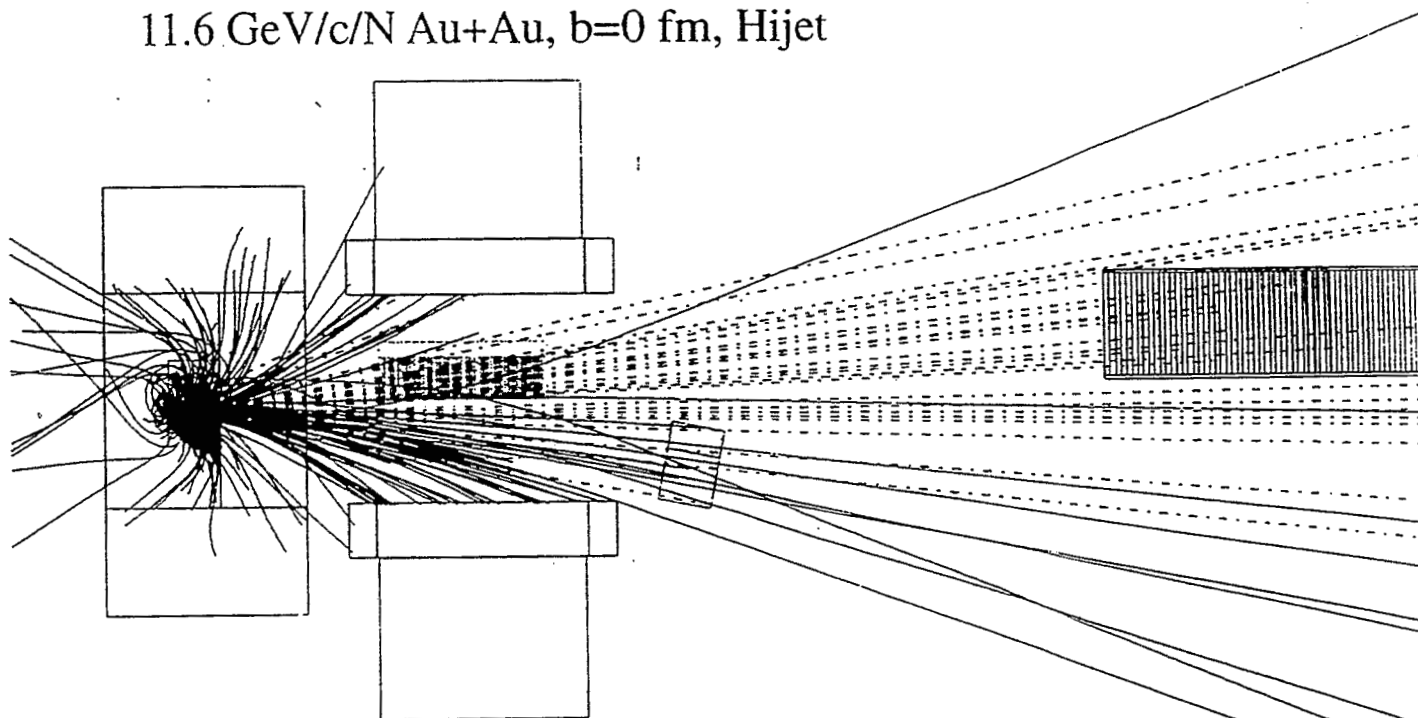


Figure 25: Reconstructed Au + Au event projected onto the pad plane.

E896 Baseline Configuration



11.6 GeV/c/N Au+Au, $b=0$ fm, Hijet



Almost all of the charged primaries are swept away from the DDC
 Enhances the probability that a measurable DDC track is a
 daughter of a neutral primary...
 High efficiencies for H, Λ , anti- Λ , and Ks reconstruction...

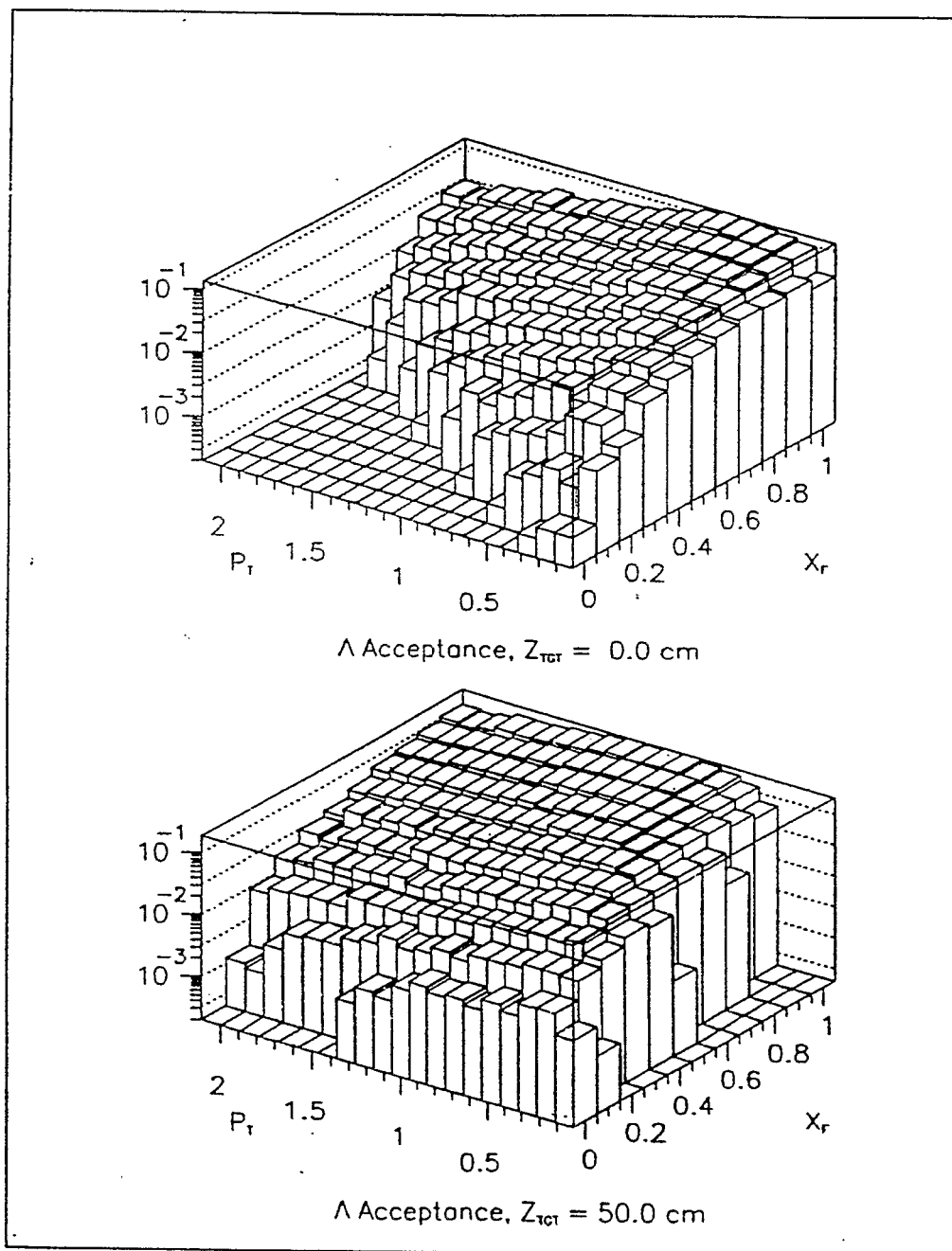


Figure 11.

Experiments at the AGS

Brian Cole

Columbia University

Λ Measurements in E910

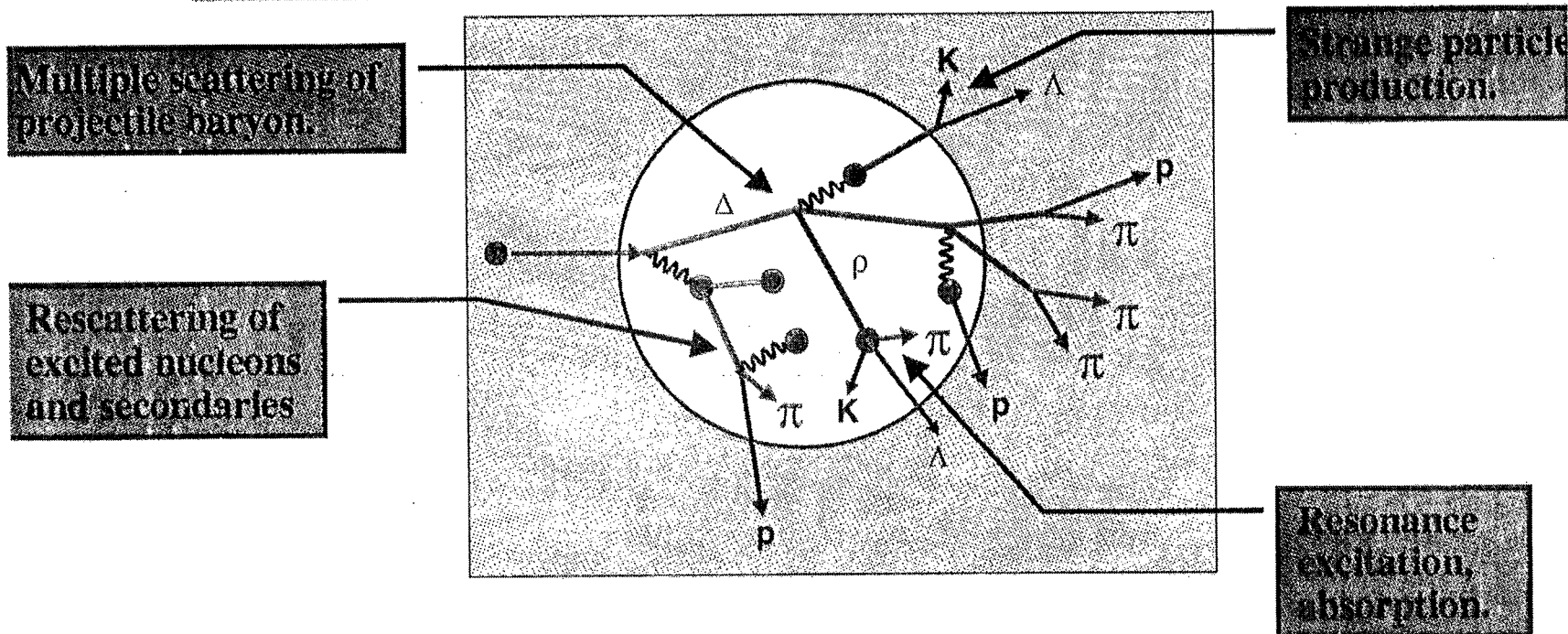
Brian A.Cole, H. Hiejima, X.Yang, D. Winter
Columbia University

RHIKEN Theory Workshop
November 11, 1997

Outline

- 1. E910 Goals**
- 2. Description of Experiment**
- 3. E910 Data & Analysis**
- 4. Λ , K_s Measurement**
- 5. Results**
- 6. Future E910 analysis**
- 7. Measuring Λ 's in Au+Au Collisions**

E910 - Goals of the Experiment



Goals of the Experiment

- Provide quantitative understanding of microscopic processes that take place during heavy-ion collisions.
- Identify mechanism responsible for **strangeness enhancement** in heavy-ion collisions
- Search for short-lived H dibaryon

E910 - Collaboration

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Brookhaven National Laboratory

B.A. Cole, I. Chemakin, H. Hiejima, M. Moulson,

D. Winter, X. Yang, W.A. Zajc, Y. Zhang

Columbia University, Nevis Laboratories

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Kent State University

G. Rai

Lawrence Berkeley National Laboratory

V. Cianciolo, E. Hartouni, M. Kreisler, R. Soltz,

M.N. Namboodiri, J. Thomas

Lawrence Livermore National Laboratory

A.D. Frawley, N. Maede

Florida State University

M. Gilkes, R.L. McGrath, Y. Torun

SUNY, Stonybrook

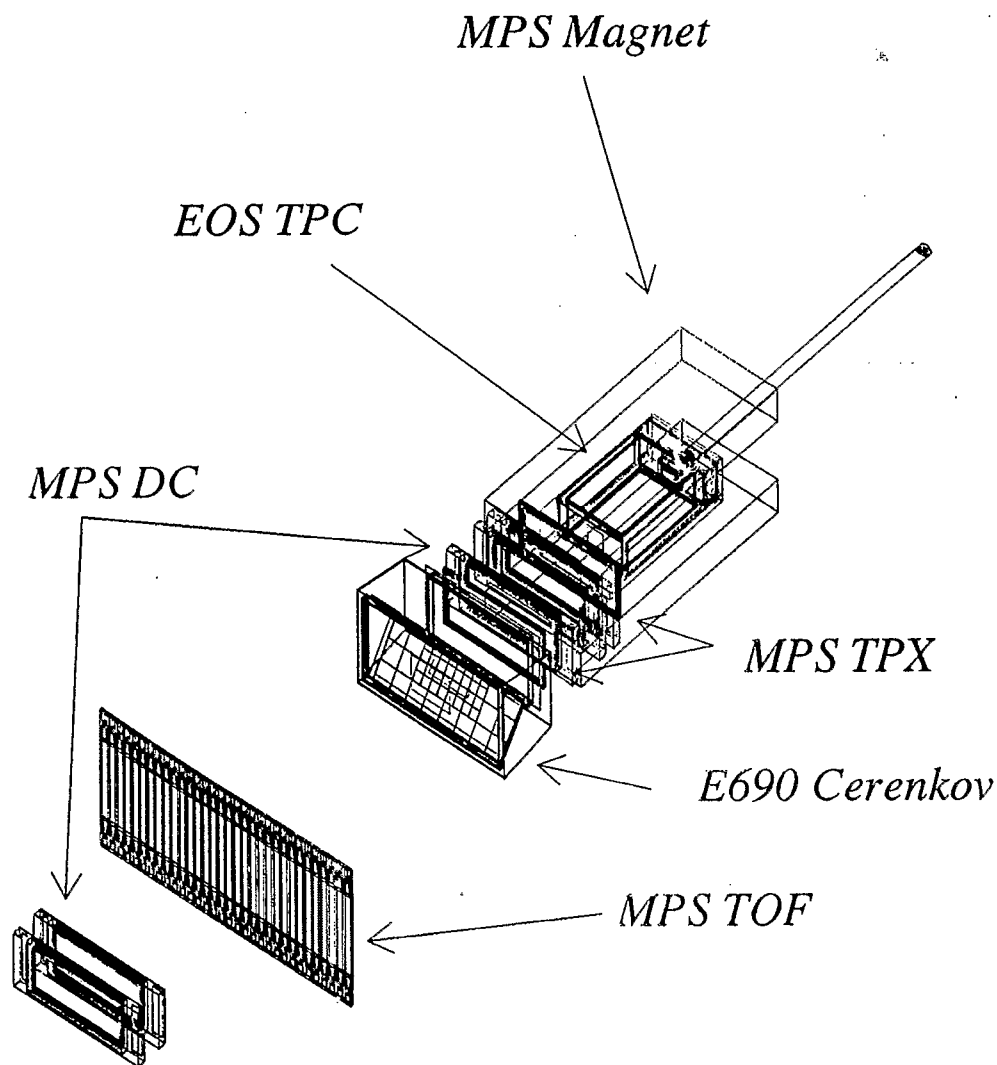
S. Mioduszewski, D. Morrison, K. Read, S. Sorensen

University of Tennessee

J. H. Kang, Y. H. Shin

Yonsei University

E910 @ AGS - Experimental Setup

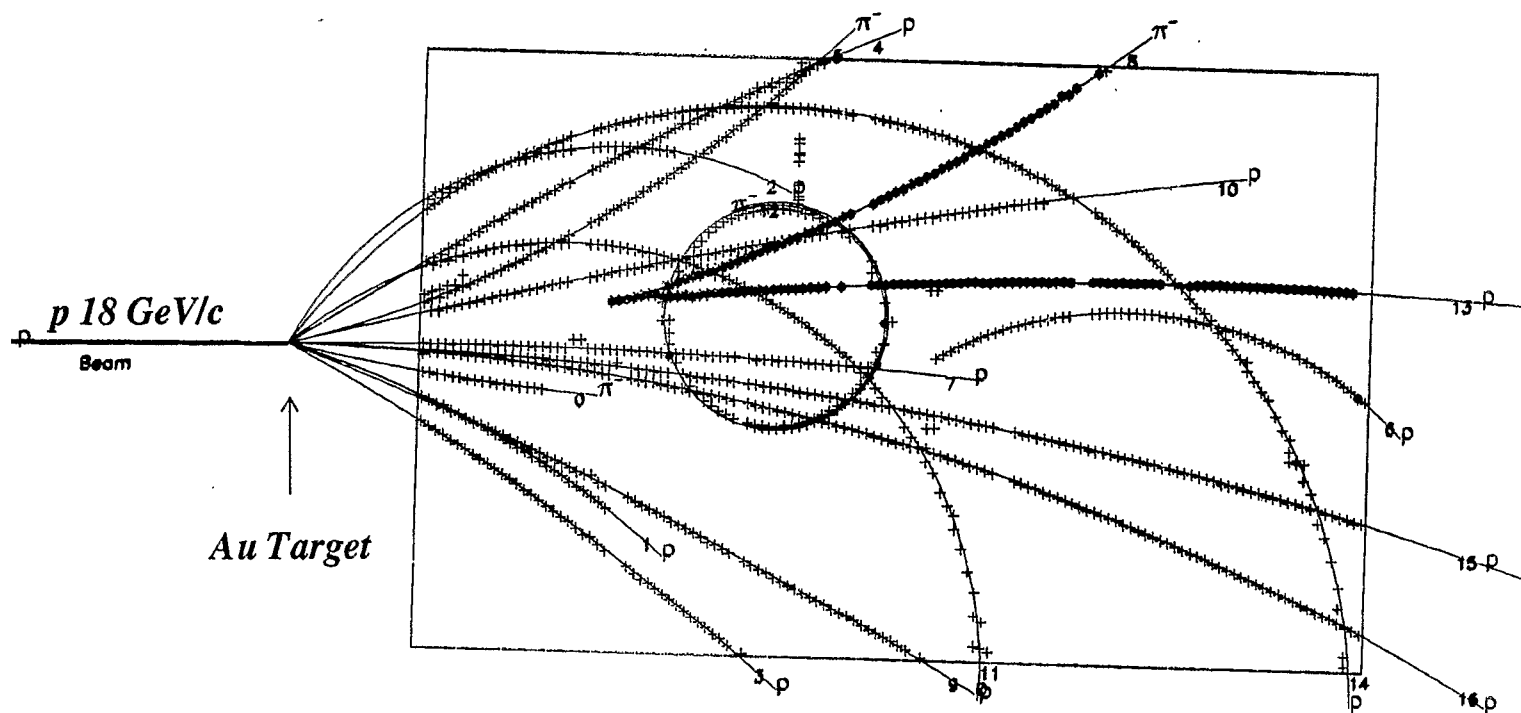


Details

- “Opportunistic” expt. using mostly existing equipment.
- Nearly complete coverage of final state (with PID).
- Installed in the A1 (MPS) secondary beam line.
- Ran for 4 months during the spring 96 AGS run.
- Used Be, Cu, Au and U targets.
- Ran at 6, 12, 18 GeV/c incident proton momenta.
- Collected ≈ 15 million min. bias and “central” triggers.

E910 - Event Display Picture

A Typical Λ Decay Event in EOS TPC

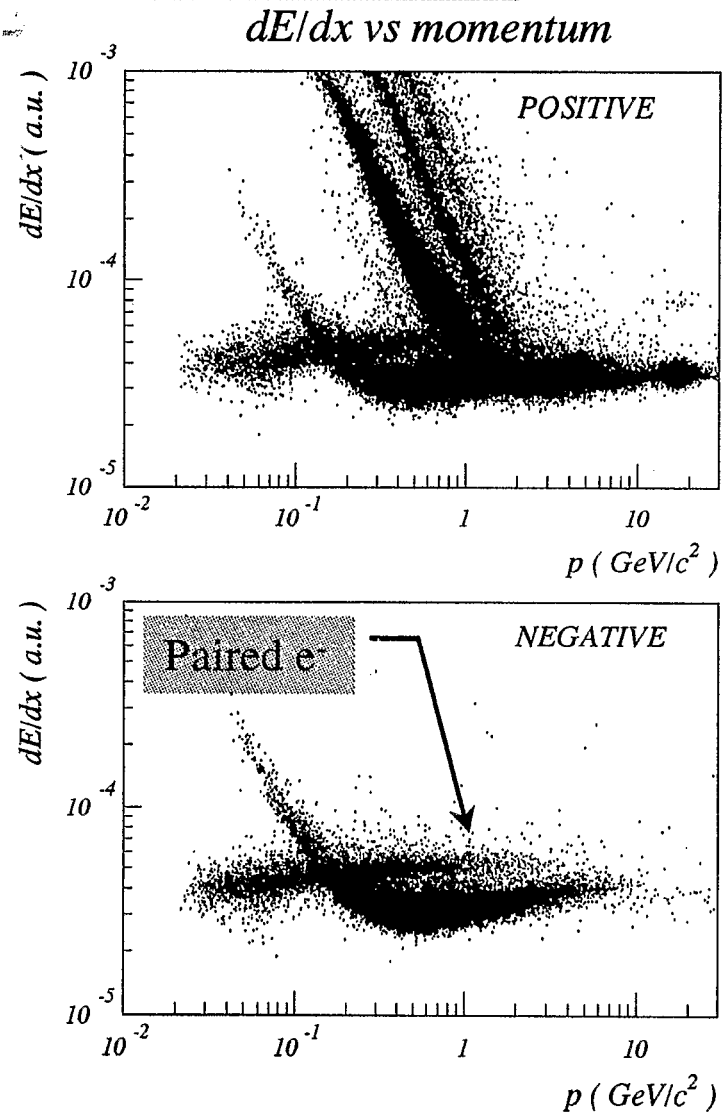
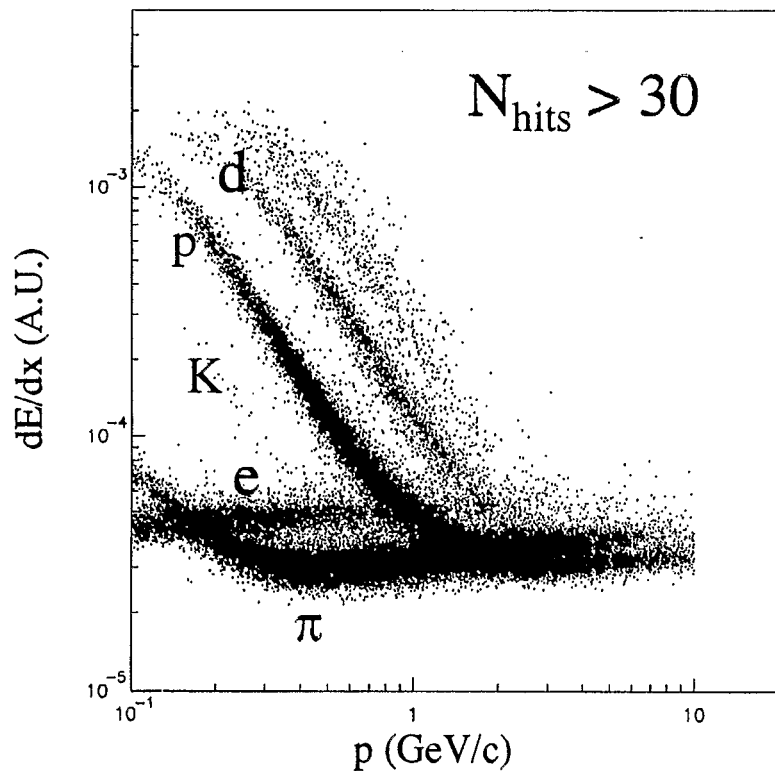


- Target $\approx 20 \text{ cm}$ upstream of TPC \Rightarrow tracks well separated at entrance
- Λ decays in TPC easily identified
- TPC resolution: 0.3 mm in x (?), 0.7 mm in y
 \Rightarrow Λ decays before TPC also easily identified

E910 - Particle Identification

dE/dx Identification

- Obtain up to 128 ΔE samples/track
- Use $\langle \Delta E / \Delta X \rangle$ vs p for particle ID.
- Pair conversion electrons to reduce electron contamination.

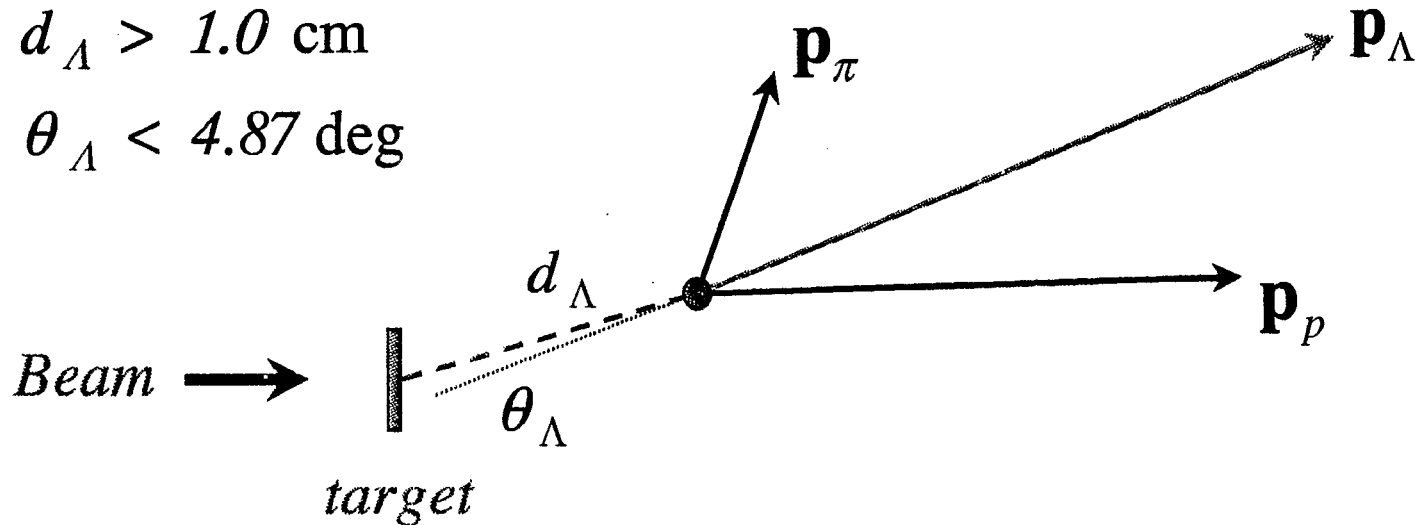


E910 - V^0 Reconstruction

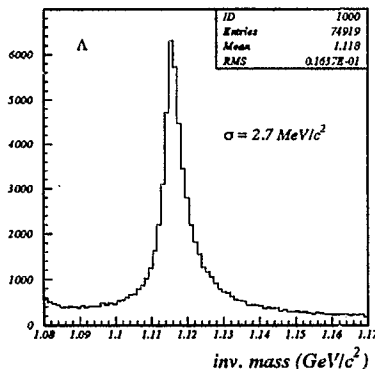
- $\Lambda \rightarrow p\pi^-$ with 63.9% BR
- $K_s \rightarrow \pi^+\pi^-$ (68.6% BR)
- Particle identification:
 - 3σ dE/dx cuts on proton and π^- tracks.
 - e^+e^- pairing to remove electrons

A Schematic View of Λ Reconstruction

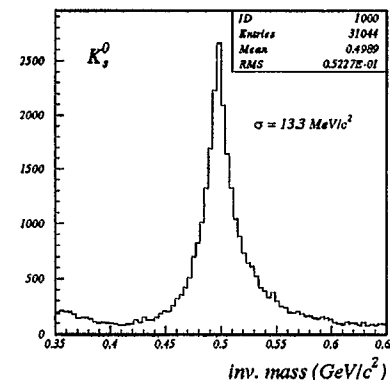
- $d_\Lambda > 1.0$ cm
- $\theta_\Lambda < 4.87$ deg



E910 - Strange Particle Production

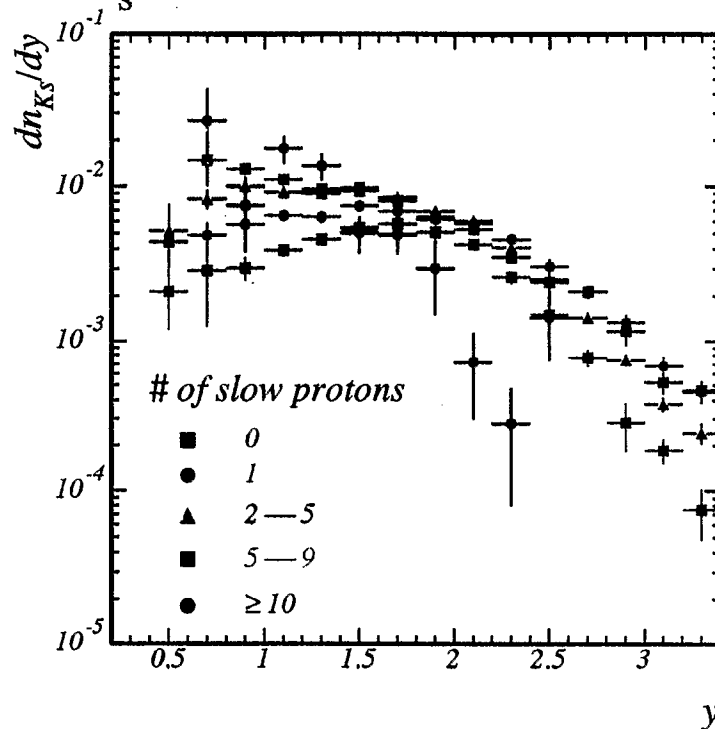
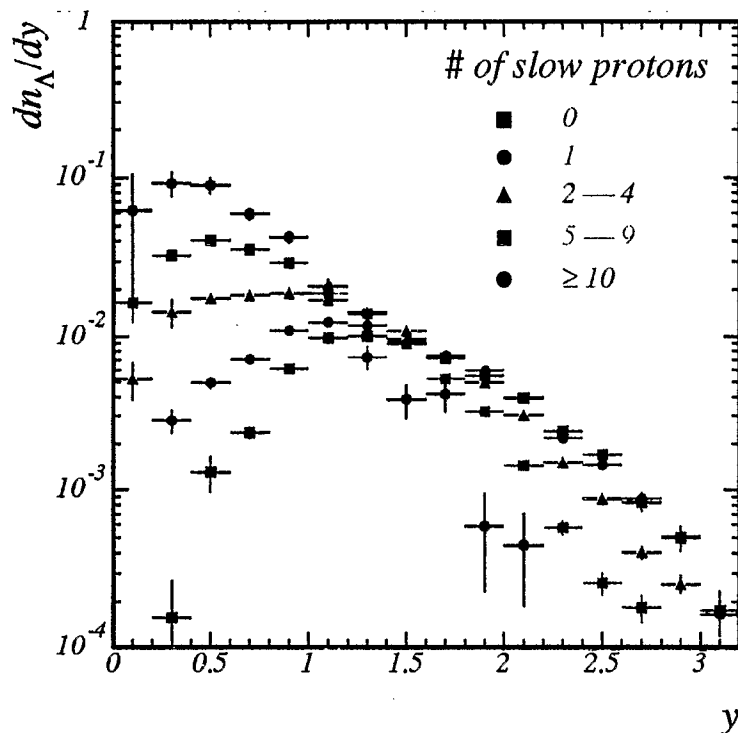


First detailed study of strange
 particle spectra vs centrality in
 p-A collisions
 (18 GeV/c p+Au collisions)
 Analysis by X. Yang - Columbia



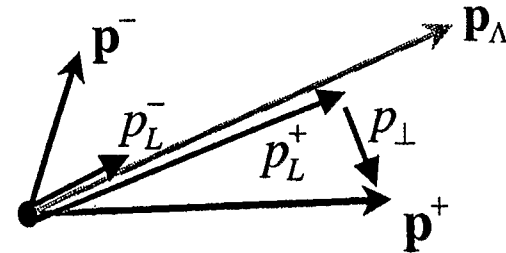
$\Lambda \rightarrow p\pi^-$

$K_s \rightarrow \pi^+\pi^-$

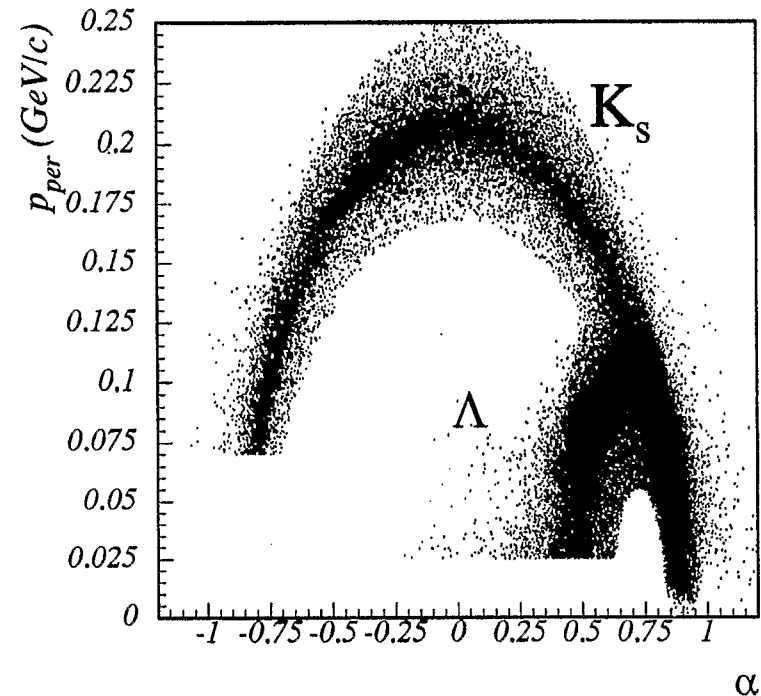
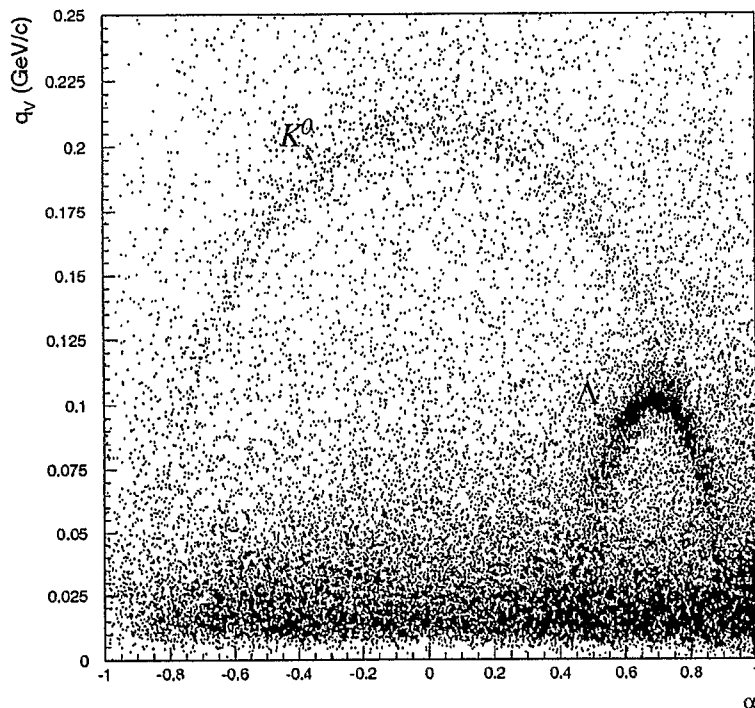


E910 - Armenteros Plot

- Method of “identifying” V^0 's without PID.
- Plot of p_\perp vs $\alpha \equiv (P_L^+ - P_L^-) / (P_L^+ + P_L^-)$
- $\alpha = c_1 \cos \theta_{cm} + c_2$, $p_\perp = p^* \sin \theta_{cm}$,
- For Λ , $\alpha = (0.17/\beta_\Lambda) \cos \theta_{cm} + 0.7$



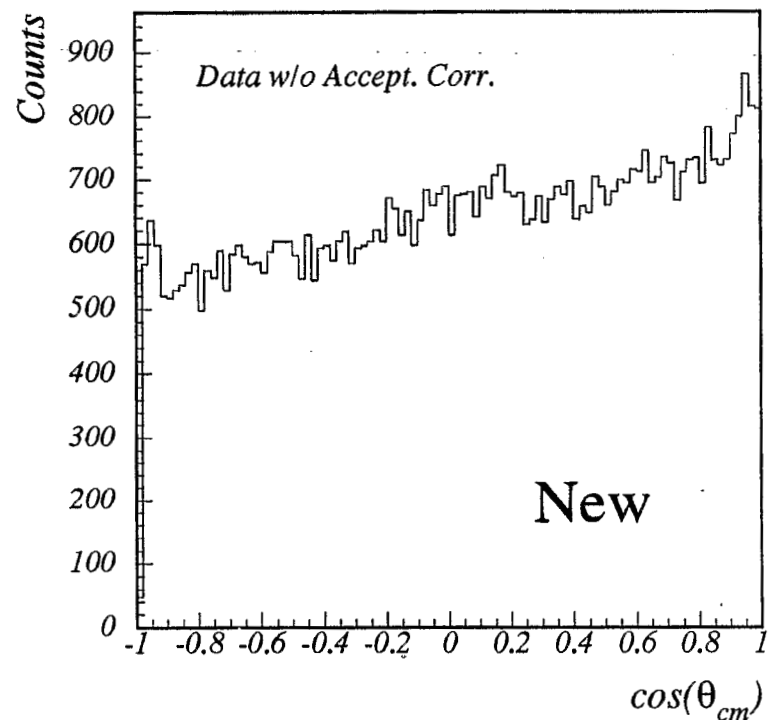
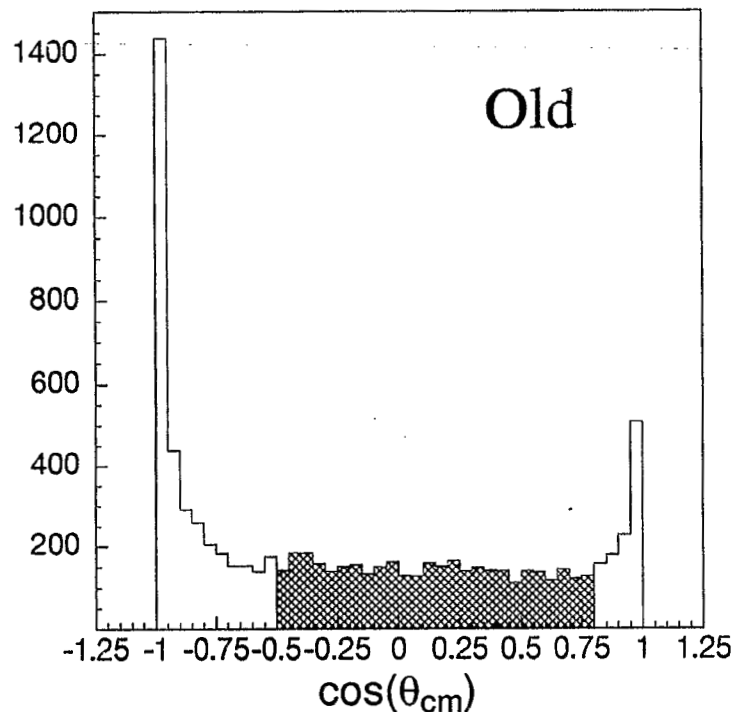
“New” analysis w/ dE/dx PID,
more statistics and “loose” M_{inv} cut



E910 - Decay directions

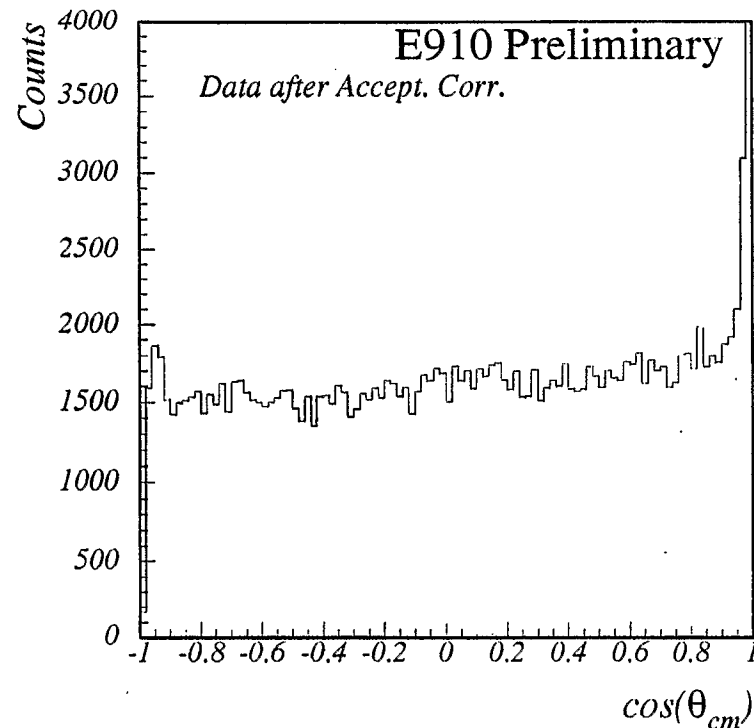
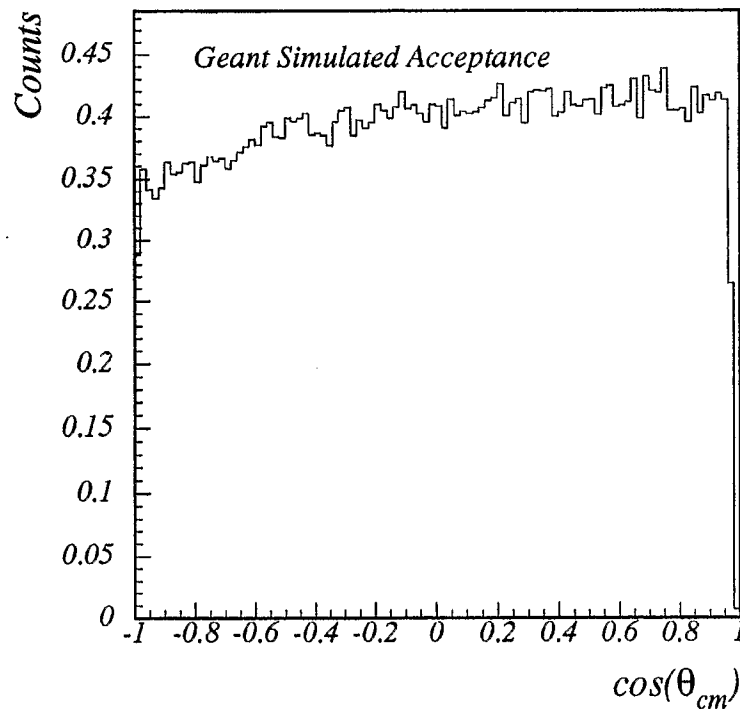
Compare “old” and “new” analyses

- Old analysis has large backgrounds at large $|\cos \theta_{\text{cm}}|$
- New analysis has much less background (**particle ID !!**)
- New analysis shows θ_{cm} dependent acceptance



E910 Decay directions (2)

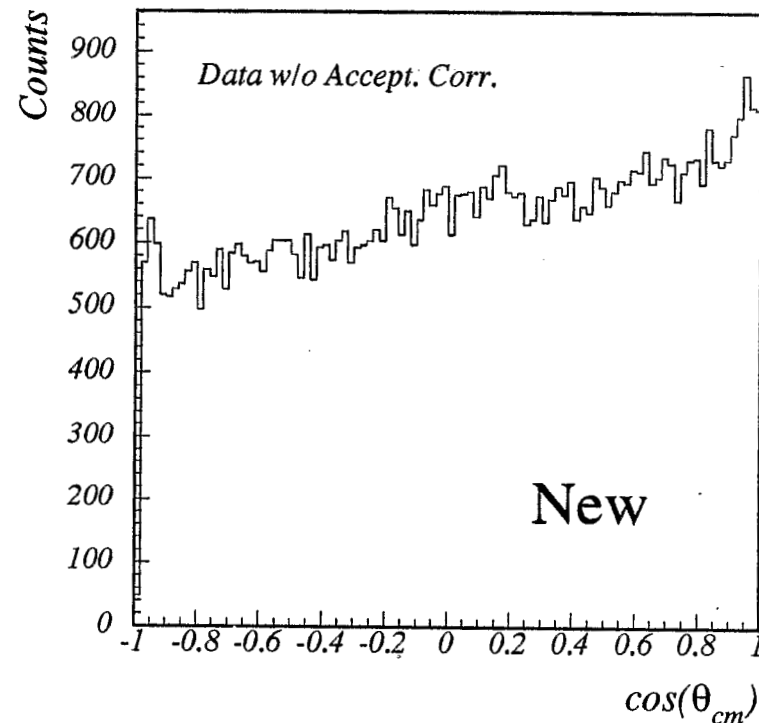
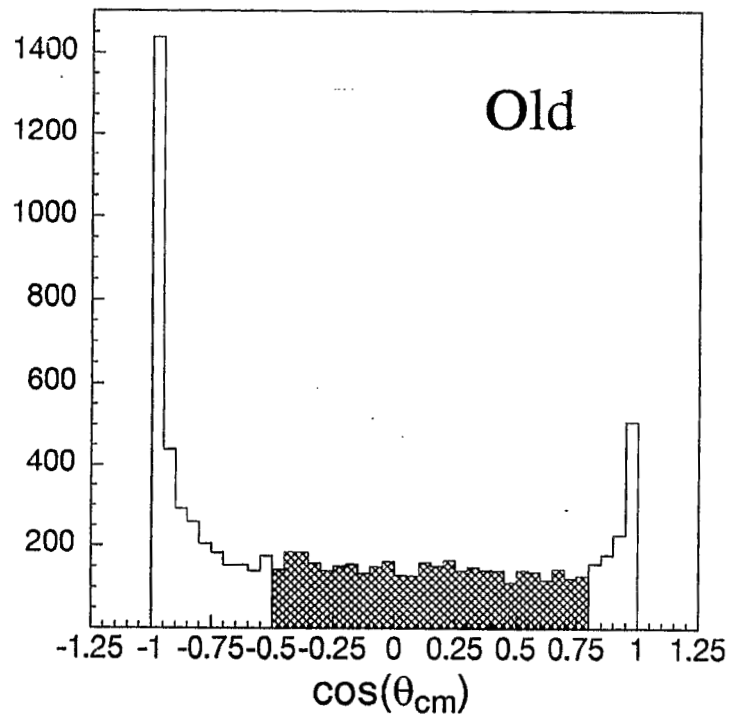
- As expected, acceptance shows $\cos \theta_{\text{cm}}$ dependence.
- After acceptance correction, see excess near $\cos \theta_{\text{cm}} = 1$.
 - Residual K_s contamination
 - Real problem for polarization measurements (?)



E910 - Decay directions

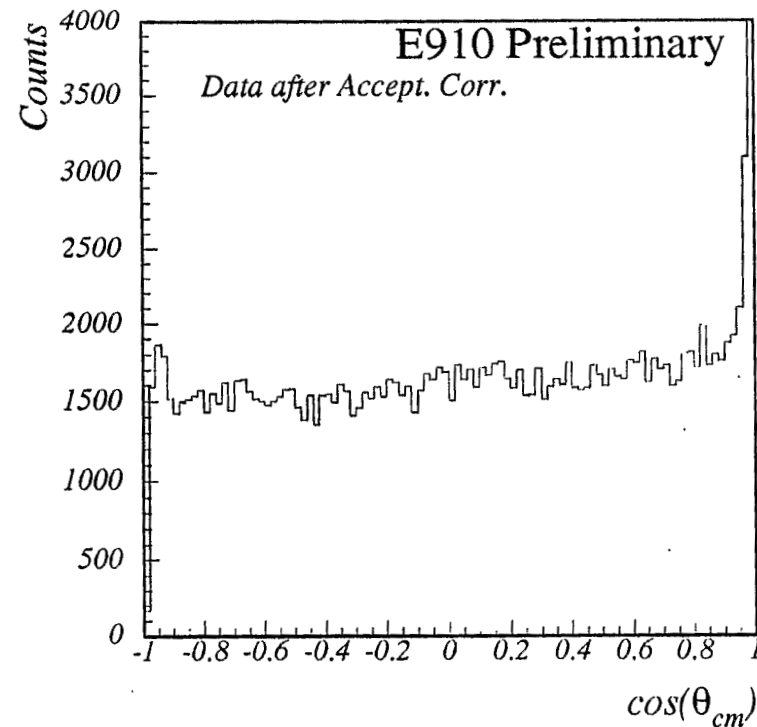
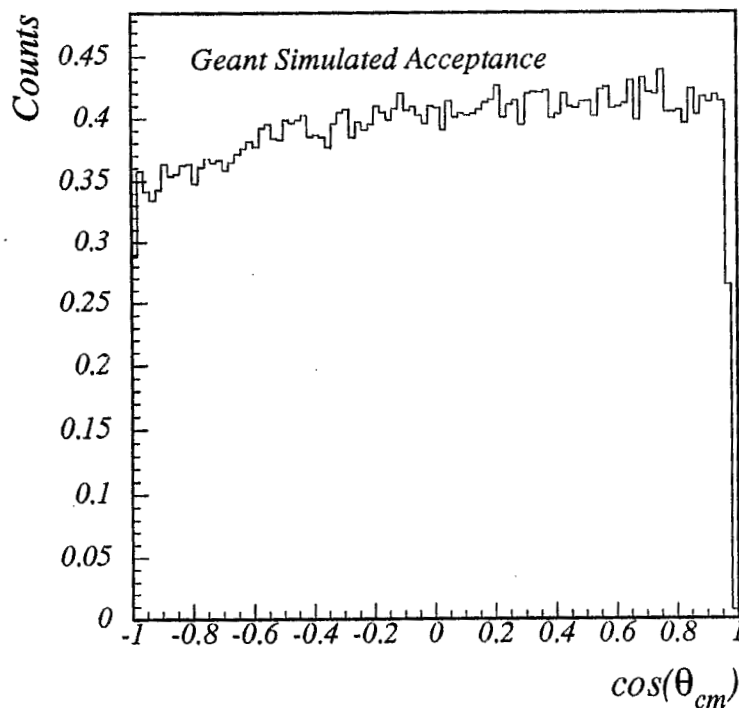
Compare “old” and “new” analyses

- Old analysis has large backgrounds at large $|\cos \theta_{\text{cm}}|$
- New analysis has much less background (**particle ID !!**)
- New analysis shows θ_{cm} dependent acceptance



E910 Decay directions (2)

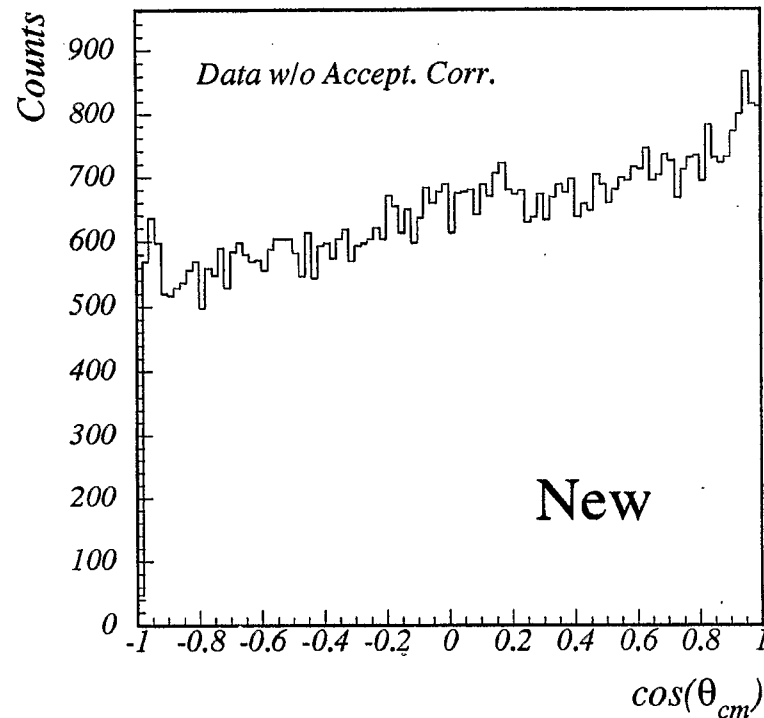
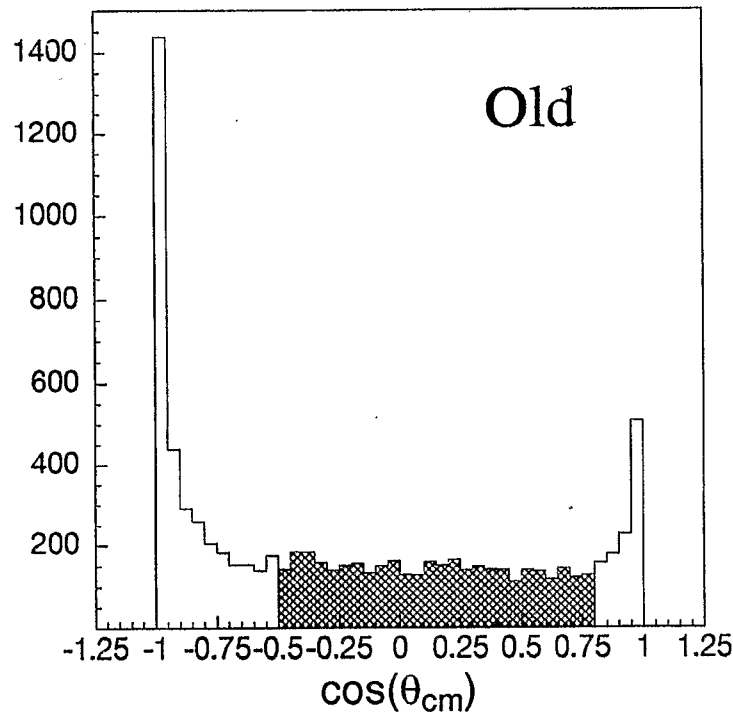
- As expected, acceptance shows $\cos \theta_{cm}$ dependence.
- After acceptance correction, see excess near $\cos \theta_{cm} = 1$.
 - Residual K_s contamination
 - Real problem for polarization measurements (?)



E910 - Decay directions

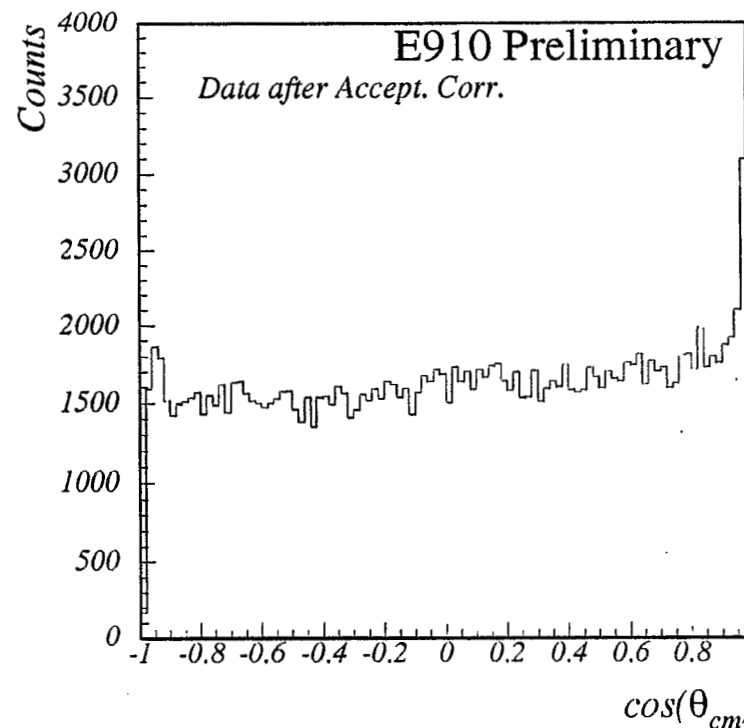
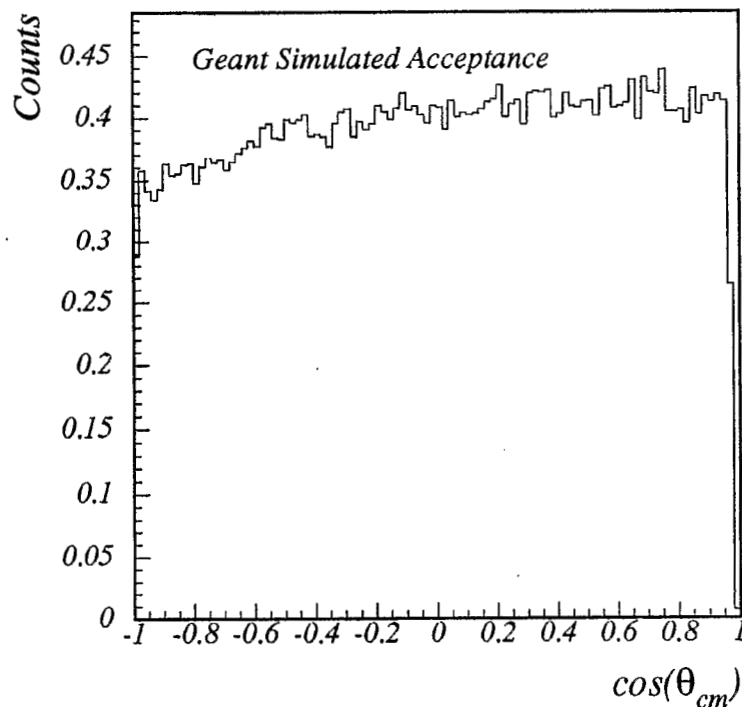
Compare “old” and “new” analyses

- Old analysis has large backgrounds at large $|\cos \theta_{\text{cm}}|$
- New analysis has much less background (**particle ID !!**)
- New analysis shows θ_{cm} dependent acceptance

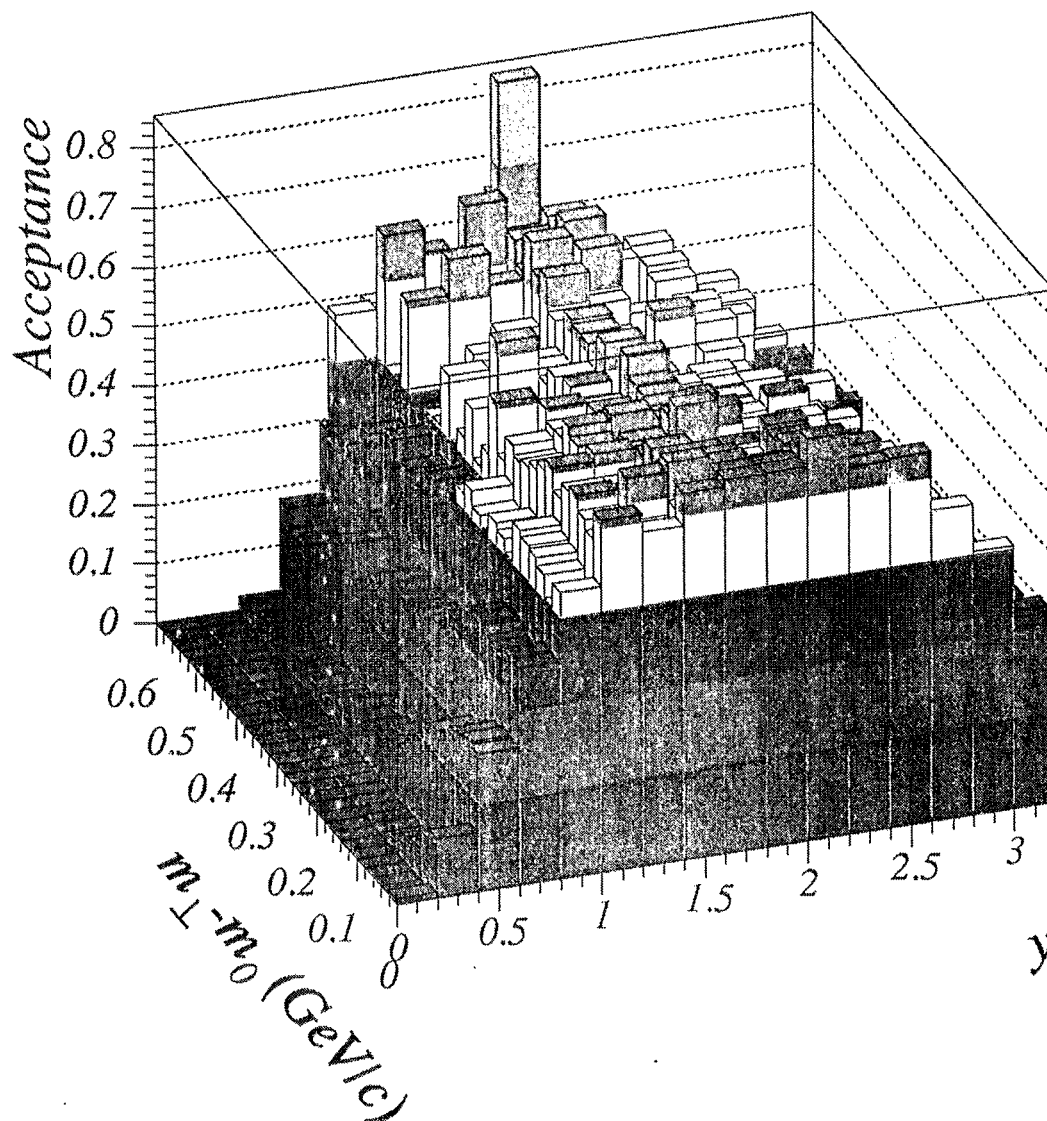


E910 Decay directions (2)

- As expected, acceptance shows $\cos \theta_{\text{cm}}$ dependence.
- After acceptance correction, see excess near $\cos \theta_{\text{cm}} = 1$.
 - Residual K_s contamination
 - Real problem for polarization measurements (?)



E910 - Lambda Kinematic Acceptance

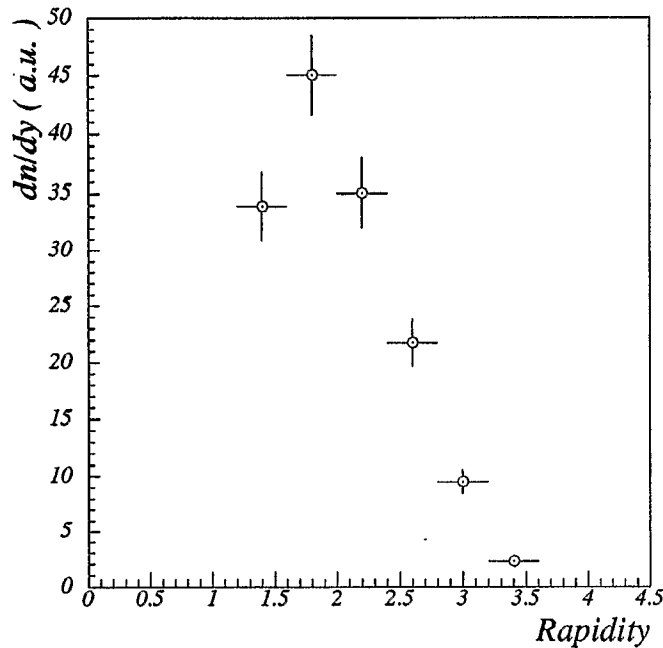


- Result of Geant Monte-Carlo.
- Used same cuts as in data analysis.
- Includes effect of BR
 - maximum = 0.64
- Losses at low y
 - Decay distance cut
 - Geometric accept.
 - Mult. Scattering
- Losses at large y
 - TPC Two-track resolution
 - Decays past TPC
 - p resolution

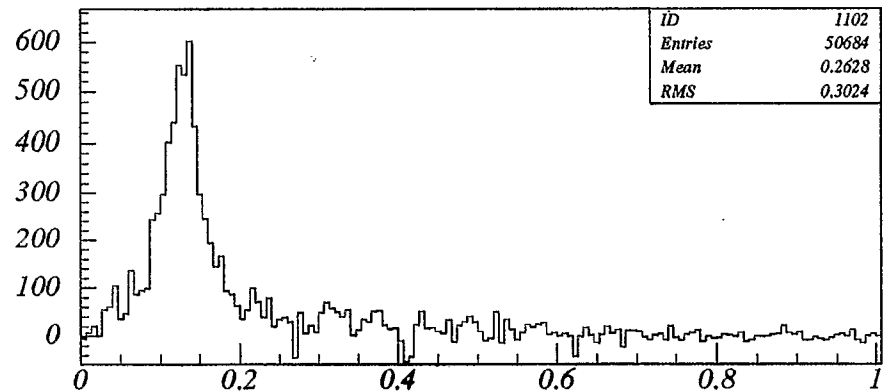
E910 γ , π^0 reconstruction

- Reconstruct e^+e^- pairs from γ conversions. Find $\pi^0 \rightarrow \gamma\gamma$

dndy vs rapidity for π^0

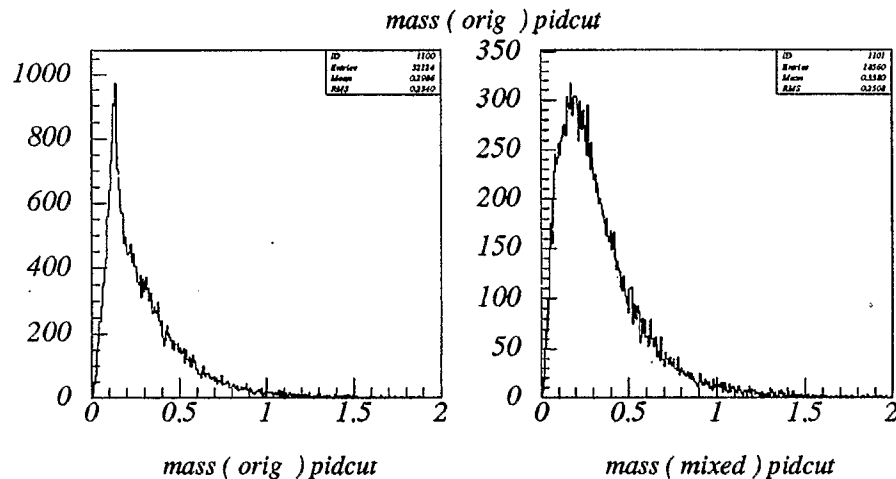


Analysis by H. Hiejima, Columbia



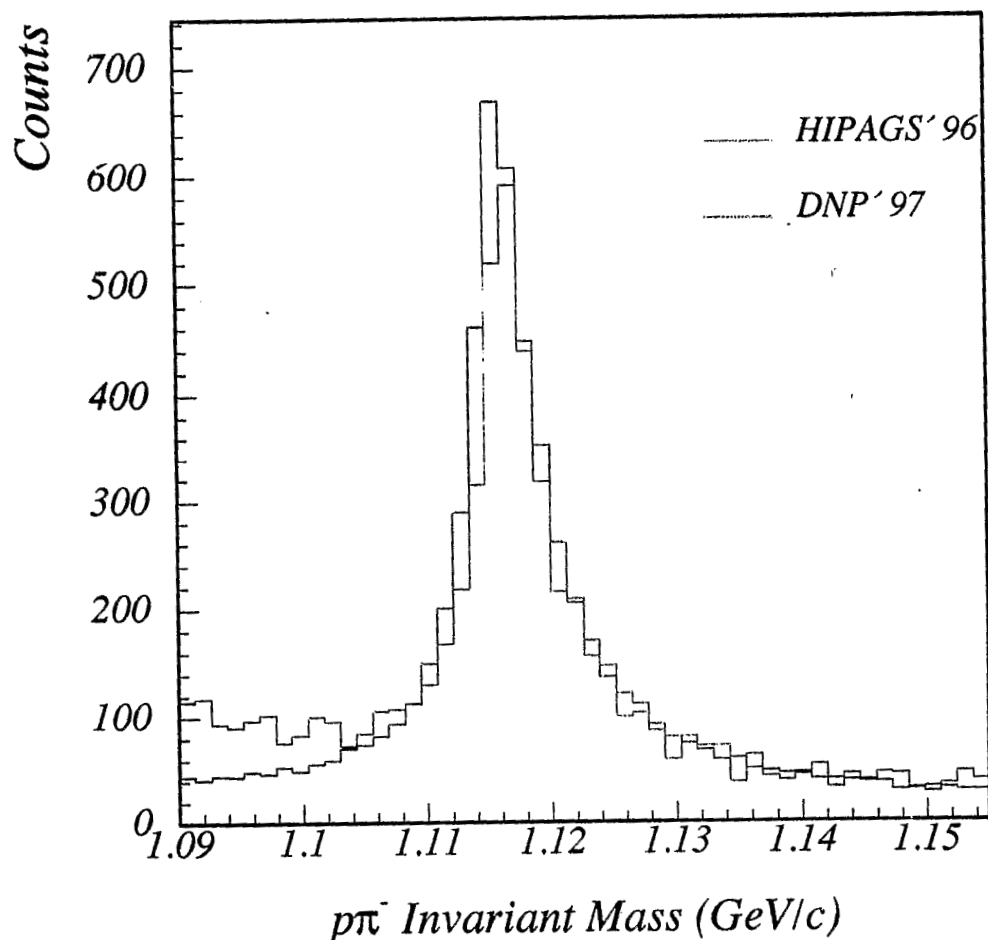
Opens many possibilities:

- $\omega \rightarrow \pi^+ \pi^- \pi^0$
- $\Sigma^0 \rightarrow \Lambda \gamma$
- $K^* \rightarrow K^\pm \pi^0$
- $\eta \rightarrow \gamma\gamma ??$



E910 - Lambda Inv. Masses

Compare “old” and “new” analyses

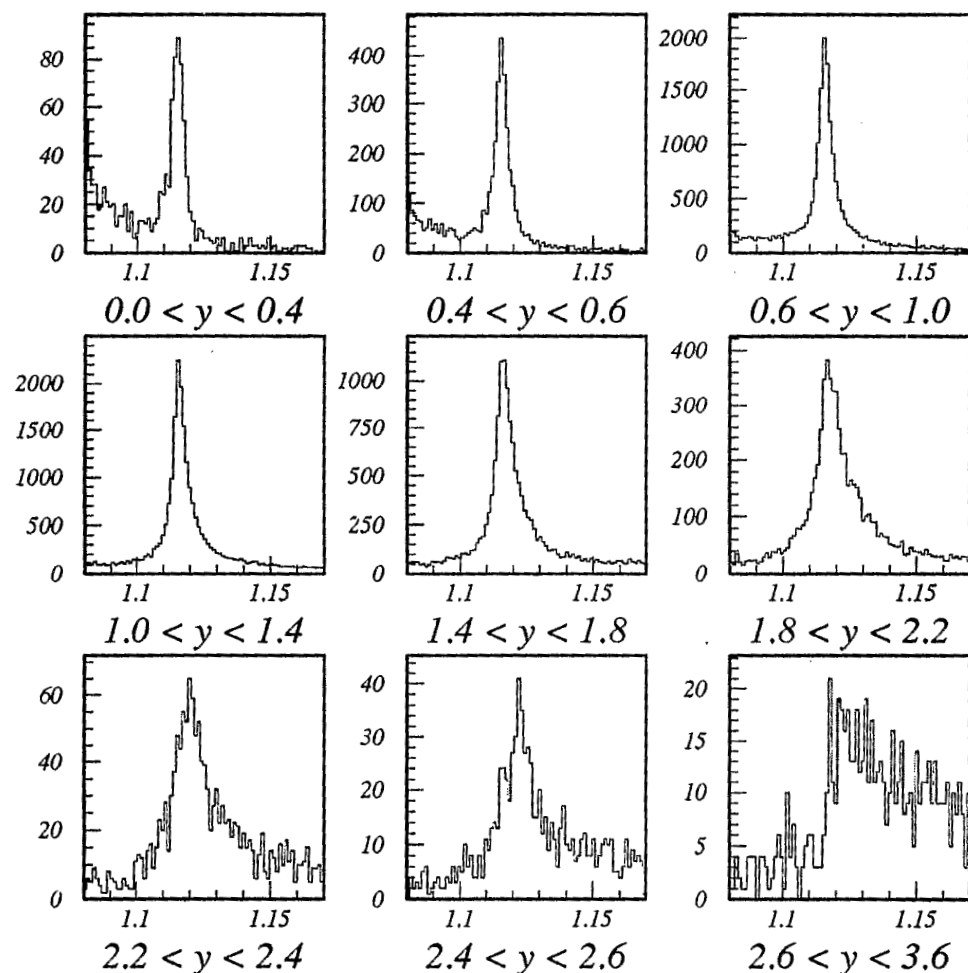


Observations

- New analysis has better S/N even w/o cut on θ_{cm} .
- New analysis has less background at low M_{inv} .
- Xihong:
 - Low M_{inv} bg mostly from fake V^0 's.
 - High M_{inv} bg mostly from K_s .
- Difficult to “see” K_s background in the inclusive distribution.

E910 - Lambda Mass vs Rapidity

Λ Invariant Mass at Different Rapidity Region

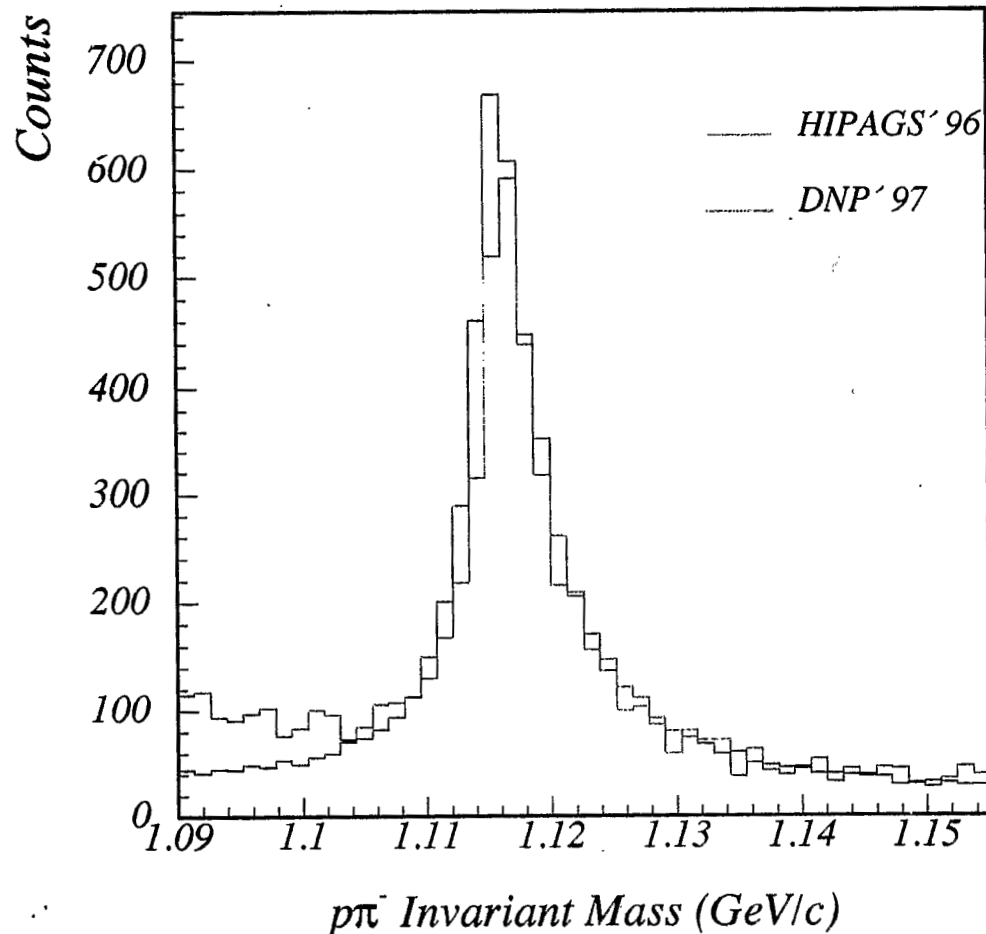


Observations

- See “fake” V^0 background at low rapidity (expected).
- See larger K_s contribution at high rapidity.
- See effect of worsening TPC momentum resolution
- Where are the Λ 's at $y > 2.6$
 - Most likely due to stopping
 - “Efficiency” loss
 - Unknown problems ??
- Λ polarization at large y problematic w/o additional particle ID due to K_s
- Note: currently not using relativistic rise in dE/dx .

E910 - Lambda Inv. Masses

Compare “old” and “new” analyses

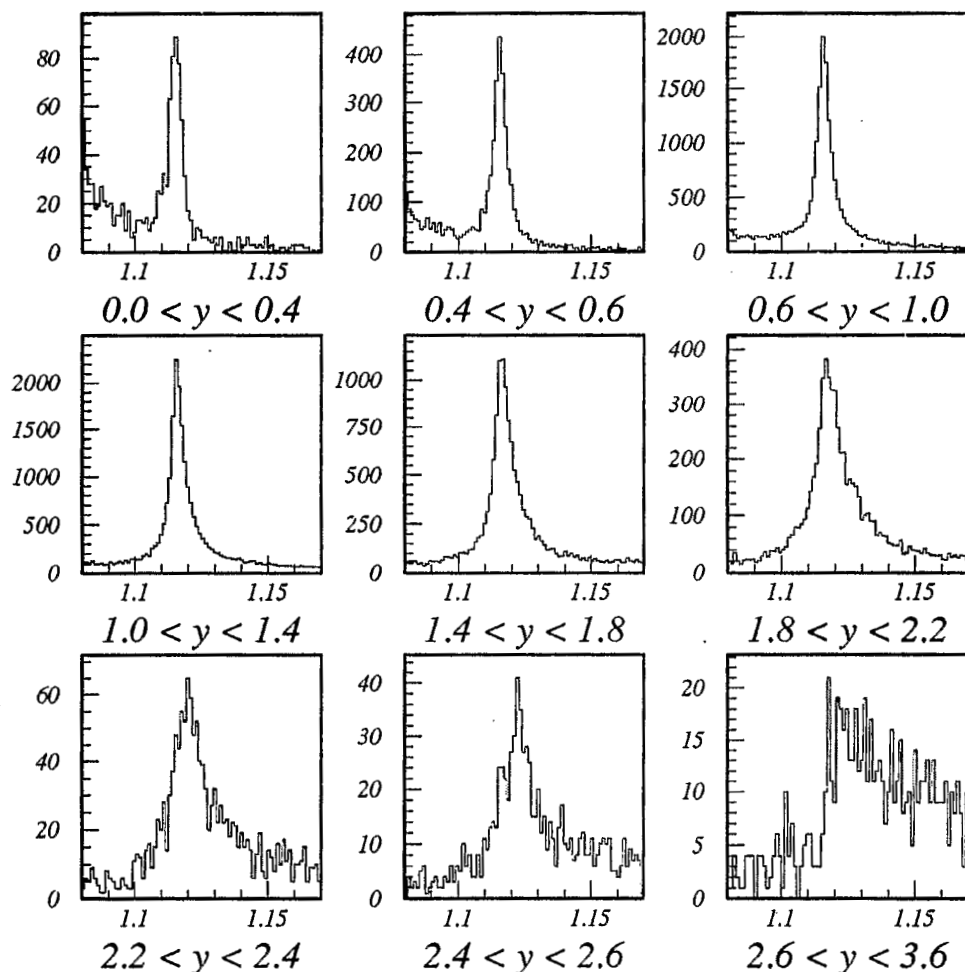


Observations

- New analysis has better S/N even w/o cut on θ_{cm} .
- New analysis has less background at low M_{inv} .
- Xihong:
 - Low M_{inv} bg mostly from fake V^0 's.
 - High M_{inv} bg mostly from K_s .
- Difficult to “see” K_s background in the inclusive distribution.

E910 - Lambda Mass vs Rapidity

Λ Invariant Mass at Different Rapidity Region



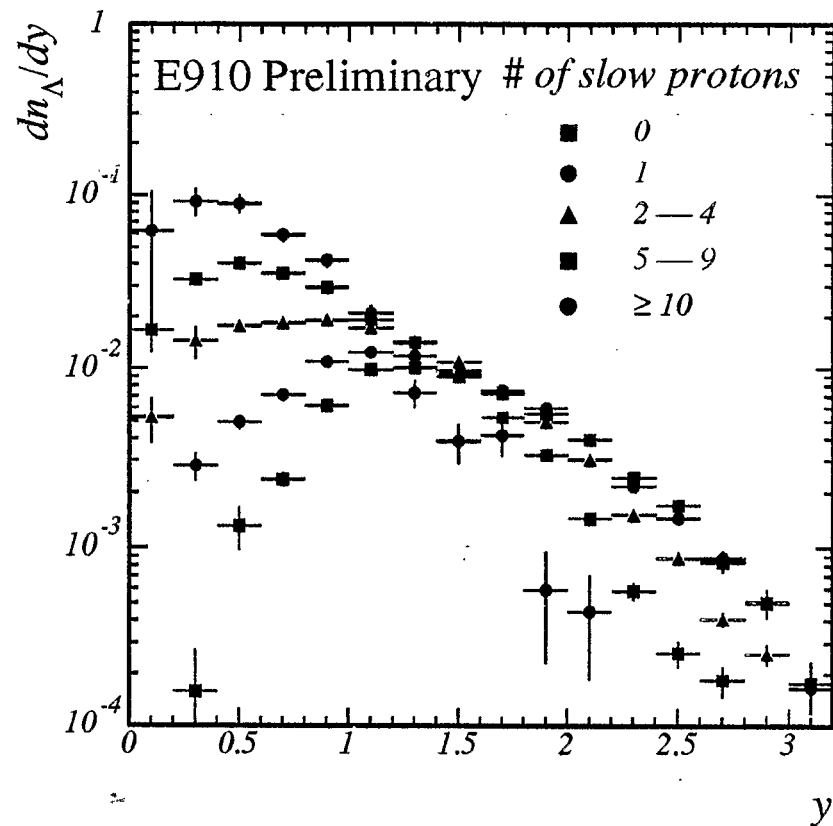
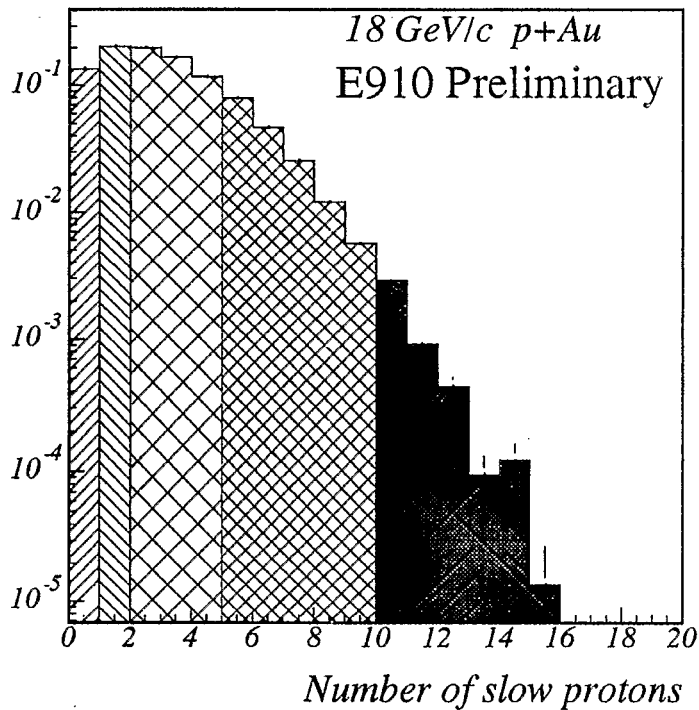
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E910 - Lambda Spectra

- “Centrality” is difficult to measure in proton-nucleus collisions.
- Previous p-A experiments: characterize centrality with # slow protons
- “Rule of thumb”, number of primary collisions, $\nu = \sqrt{N_{slow}}$
- First detailed study of centrality dependence of Λ production in p-A collision.

Slow proton has $p < 1.2$ GeV/c



E910 - Future of Lambda Analysis

Detector Analysis

- TPC
 - Improvements in “hit-finding” for close tracks.
- Particle Identification
 - Currently tuning up TOF and Cherenkov identification
 - Relativistic rise dE/dx helpful for tracks that miss Cherenkov.
- Downstream tracking
 - Improved ($\times 10$) momentum resolution at high p .
 - Eventually will reconstruct Λ decays at end of or after TPC.

Physics Analysis

- Improved Λ reconstruction cuts
- Other targets (Be, Cu), beam momenta (12 GeV/c)
- Lambda polarization measurement
- Λ Λ correlation analysis

Lambda Polarization Correlations

Main issues (guesses - no real experience)

- Acceptance
 - Because it comes in squared for pairs
 - ⇒ e.g. even if $dN/dy = 10$, if $A = 0.01$, pair rates are low
 - ⇒ even with “large” geometric coverage E910 has $\langle A \rangle = 0.4$.
 - More important (critical):
 - ⇒ Restricted geometric accept. \Rightarrow restricted polarization accept.
 - ⇒ Induces automatic correlation in Λ polarization.
 - ⇒ Analogy of “residual” correlation in HBT (?)
 - ⇒ Difficult to control systematic errors at few % level ?!!.
- Contamination/feed-down
 - Because it comes in “squared” for pairs (fraction of true $\Lambda\Lambda$ pairs).
 - So many sources: e^- faking π , fake V^0 's, K_s , $\Sigma^0 \rightarrow \Lambda\gamma$, Ξ^- , secondary interactions, ...
 - Effects may be worse (e.g. K_s) where acceptance is better (high y).

Lambda Polarization Correlations (2)

How to design expt. from scratch (starting point) ?

- Use large-acceptance detector (rate, ...)
- Measure at moderate to high rapidity
 - Angular acceptance is better for given geometric size.
 - Get downstream from target & away from the junk.
 - Less multiple scattering, energy loss, ...
 - Need good tracking resolution, particle ID.
- Make azimuthal acceptance uniform
 - “Guaranteed” to have bias in θ_{cm}
 - Look only for correlations azimuthally ??
- Start with/use asymmetric collisions
 - Demonstrate no effect in control measurement (e.g. O+Au)
 - Less uninteresting corona in (e.g.) Ag+Au than Au+Au ?
 - For central interactions, Λ at mid-rapidity and above must come from high baryon density region.



RIKEN BNL RESEARCH CENTER

SYMPOSIUM ON COLOR SUPERCONDUCTIVITY, INSTANTONS, AND PARITY (NON?) CONSERVATION AT HIGH BARYON DENSITY

NOVEMBER 11, 1997

PHYSICS DEPARTMENT SEMINAR ROOM

AGENDA

09:00 -- 09:30	Krishna Rajagopal (MIT)	
		QCD at Finite Baryon Density: Nucleon Droplets and Color Superconductivity
09:30 -- 10:00	Mark Alford (IAS)	
10:00 -- 12:00	Thomas Schäfer (INT)	Instanton Dynamics
11:00 -- 12:00	Yang Pang (Columbia)	Parity Violation Observables in AA
01:00 -- 01:30	Nicholas Samios (BNL)	Lambda Polarization Measurements
01:30 -- 02:15	Brian Cole (Columbia)	Experiments at the AGS
02:30 -- 05:00	Discussions	

RIKEN BNL Center Workshops

Title: Physics with Parallel Processors
Organizers: R. Mawhinney/S. Ohta
Dates: April/May 98 (tentative)
Title: RHIC Spin Physics
Organizers: G. Bunce/M. Tannenbaum
Dates: April 27-29, 1998
Title: Quarkonium Production in Relativistic Nuclear Collisions
Organizer: D. Kharzeev
Dates: Sept. 28-Oct. 2, 1998
Title: Dynamics of Chiral Fields in Nuclear Collisions
Organizer: D. Rischke
Date: Week of April 27th
Title: QCD Spectroscopy
Organizers: M. Pennington/W. Marciano
Dates: TBA
Title: Polarized Parton Distributions and Event Generators
Organizers: TBA
Date: Summer (tentative)
Title: QCD Vacuum and Phase Transitions
Organizer: E. Shuryak
Date: Early spring
Title: Semiclassical Fields in QED and QCD
Organizers: A. Baltz/D. Rischke/L. McLerran
Date: Early fall
Title: Quantum Fields in and out of Equilibrium
Organizers: R. Pisarski/H. de Vega
Date: Week of October 26th
Title: Many-Fermion Systems ... (Bielefeld follow-up)
Organizers: M. Creutz/M. Gyulassy
Date: November 1998
Title: Many-Fermion Systems at Finite Densities
Organizers: T. Blum/M. Creutz
Date: early '99
Title: Spin Physics Mini-Workshops
Organizers: N. Samios/R. Jaffe
Date: Quarterly

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RIKEN BNL RESEARCH CENTER

COLOR SUPERCONDUCTIVITY, INSTANTONS AND PARITY (NON?)-CONSERVATION AT HIGH BARYON DENSITY

NOVEMBER 11, 1997



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*Nuclei as heavy as bulls
Through collision
Generate new states of matter.
T. D. Lee*

Speakers:

M. Alford
K. Rajagopal
T. Schäfer

B. Cole
Y. Pang

M. Gyulassy
N. P. Samios

Organizer: M. Gyulassy